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New Trends in Stochastic Modelling for Hydrologic Data Analysis

ALI UNAL SORMAN
Department of Hydrology and Water Resources Management,
Faculty of Meteorology, Environment, and Arid Land Agriculture,
King Abdulaziz University, Jeddah, Saudi Arabia

ABSTRACT. This paper looks at monthly rainfall-runoff data series to see if the effects of any change in time scale can be identified. Monthly rainfall and runoff data in Wadi Jizan were processed to analyze for linear trends, periodicities, autoregressions and random residuals.

Possible physical causes are given for the components, some of which are found to be significant in model formulation.

Introduction

A stochastic modelling of rainfall and runoff for Wadi Jizan has been studied by the author earlier and presented as a paper in the JKAU: Met., Env., Arid Land Agric. Sci. (Vol. 3, 1992, pp. 95-107). In this issue, new trends in stochastic modelling for hydrologic data analysis with an application will be addressed using PC computer program in order to tackle time series problems for monthly hydrologic data analysis as compared to the conventional methods which are lengthy and not practical in application.

The objectives of this paper are to describe the time structure of the stochastic modelling in general and to show the results of model application in particular for rainfall-runoff data using the new trends as power spectral and analysis of variance approach. As a result, different components of time series model for monthly data are displayed in a tabular form and the percentage of variances explained by each component are presented numerically.

Model Description

A simple time series model is fitted considering three components namely; trend, periodic and autoregressive using the computer program presented recently in

Water Resources Publication by Kite (1991). The time series model represented by a linear additive components is expressed by the equation

$$Y_t = T_t + P_t + R_t$$

where Y_t is seasonal/annual time series data, T_t is a trend, P_t is a periodic and R_t is a stochastic component.

The trend component (T_t) is generally associated with changes in the structure of the time data caused by cumulative natural or man-made changes due to urbanization of a watershed over an extended period of time or construction of any man-made structure. Then time trends are separated from the other components by fitting a functional form such as linear or polynomial equation. A trend component is analyzed and removed by using a polynomial regression of the order shown in Table 1.

TABLE 1. Model components and parameters for hydrologic data at Wadi Jizan.

	Type	Description		Parameters							
				Rainfall				Runoff			
	General	$Y_t = T_t^1 + P_t^2 + R_t^3$		Mean (mm)		Std. dev (mm)		Mean		Std. dev (mm)	
				23.51		29.07		55.92		66.44	
1-	Trend Model	$T_t = a_0 + a_1 t + a_2 t^2 + \cdots + a_p t^p$		a_0	a_1	r	S_e	a_0	a_1	r	S_e
				2115.3	-1.056	0.083	36.76	-1586	0.83	0.061	91.0
2-	Periodic Model	$P_t = A_0 + \sum_{k=1}^{n/2} [A_k \cos(\ ) + B_k \sin(\ )]$		k	A_k	B_k	C_k^2	k	A_k	B_k	C_k^2
			mean	1	-4.92	-17.86	343.2	1	-35.28	-29.12	2083
			var	1	-3.50	-16.12	272.0	1	-29.38	-1687	1004
3-	Autoregressive (Markov) Model	$R_t = \sum_{j=0}^{k} \alpha_j R_{t-j} + \varepsilon_t$		k		α_j		k		α_j	
				1		0.016		1		0.089	

Periodicities are usually present at any time in the data due to changes in astronomical or hydrological cycles resulting from the earth's rotation. This type of trend is more common in hydrologic data series. Rainfall, runoff, evaporation and the other cycle elements show periodic trends with an annual and seasonal time period. This trend period can be identified using a trigonometric function or a spectral analysis. A periodic component is detected and removed using Shuster's periodogram as discussed in Matalas (1967) and Kite (1989) as shown in Table 1.

The third component is an auto-regressive element which reflects the memory effect and shows the tendency in the magnitude of an event to be dependent or independent on the previous seasonal event. Then it can be represented by a Markov type or moving average model. The stochastic component is assumed to be represented by Markov model in this study.

Once, these trends are identified and subtracted from the time series, analysis of variance and spectral analysis are carried out to indicate the percentage of the information explained by the variances of each component. In accomplishing the task, spectral analysis is made by the program after each step as listed below to indicate the presence of any remaining components.

1 – SELECT ORIGINAL DATA
2 – Spectral analysis #1
3 – DETECT AND REMOVE LINEAR TRENDS
4 – Spectral analysis #2
5 – DETECT AND REMOVE PERIODICITIES
6 – Spectral analysis #3
7 – DETECT AND REMOVE AUTOREGRESSION
8 – Spectral analysis #4
9 – ANALYSIS OF RESIDUALS

Discussion of Results

Time series and spectral analyses can be used to show the relative magnitude of components such as trends, periodicities and autoregression. The results of such analyses have been discussed for the hydrologic data of Wadi Jizan, which are collected at the outlet for runoff at SA 418 and for rainfall at the Jizan dam SA 001 in terms of possible cause of the model components found.

The components of the stochastic model presented in Table 1 are determined by the software program called TIME (Kite 1991) using monthly rainfall and runoff data of Wadi Jizan for a period of length (1959-81). The mean values are determined to be 23.51 mm for monthly precipitation depth and 55.92 m^3/sec for runoff. No other comments could be made about the different data lengths and their effects because the observed data record is not long enough to separate them into subgroups.

Spectral analysis and analysis of variance results are shown in Table 2 to present the sum of power spectral estimates for the original data and the other time series estimating linear trend, periodicity and autoregressive components in a sequence. The analysis of variance shows the total variance of the periodic series as well as the percentage of the variances explained by the means and the standard deviations.

The conclusive results from this analysis are summarized after running the programme for the data series as follows:

1 – The smoothed monthly data series for rainfall and runoff suggested that periodic variation can be represented by a Fourier Series with coefficients of A and B. A period of 12 months and a phase angle of about – 0.26 and – 0.88 months provided the highest smoothed spectral estimates for rainfall and runoff respectively.

2 – The physical meanings of the values for model parameters are:

a – For the periodic model:

A and B are the coefficients of fourier series, C_1^2 is the explained variance rep-

TABLE 2. Power spectral and variance of analysis.

Type	Form	Rainfall			Runoff		
		Parameters		Var	Power spectral parameters		Var
Original Time Series (OTS)	Linear trends	1338	1328.80	0.69	8251	8220.5	0.37
		Var of means		20.09			23.26
OTS - Linear trends (LT)	Periodicities	1328.8	758.51		8220.5	4184.1	
		Var of std. dev		22.52			25.86
OTS - LT - Periodicities	Autoregression	758.5	758.31	0.03	418.41	4140.8	0.32
Percentage of explained variance	By the model			43.33			49.81
Percentage of unexplained variance	Residual			56.67			50.19

resented by $(A^2 + B^2)/2$. Only the first harmonic terms are presented for the model in Table 1.

b – For autoregressive model:

Type AR(1) is selected where α_1 is the coefficient of the order one which represents the time dependence term and ε_t is the residual term (independent normal variable) representing the stochasticity.

3 – The explained variances for the first harmonic denoted by C_1^2 are found to be 343.2 and 2083 for the means; 272.0 and 1004 for the standard deviations of each variable respectively as shown in Table 1.

4 – Table 2 contains the estimated power spectral values with the total explained variances. The trend, periodic and markov models indicated that the overall model performance of the hydrologic data is low as 43.33% for rainfall and 49.81% for runoff. This means that there is a mean error, not explained by the model, of order 57% and 50% for rainfall and runoff respectively. These errors are due to the climatic factors caused by aridity.

5 – Most of the information is explained by the periodicity component, half of which is by the variance of the means and the other half is by the variance of the standard deviation (Table 2). This is one of the reasons why in the author's early paper (Sorman 1992), the stochastic rainfall and runoff models using conventional methods are expressed by the periodic autoregressive types using the same data of Wadi Jizan after the transformation.

6 – The remaining unexplained variance can be attributed to random fluctuations which are often a dominant source of variation in arid and semi-arid climates like in Saudi Arabia. This source of errors commonly results from environmental factors not measurable but may be characterized by introducing a probabilistic function or by a random stochastic element in the model.

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منحنى جديد في نمذجة التغير الإحصائي الزمني لتحليل البيانات الهيدرولوجية

علي أونال شورمان

قسم علوم وإدارة موارد المياه ، كلية الأرصاد والبيئة وزراعة المناطق الجافة

جامعة الملك عبد العزيز ، جدة ، المملكة العربية السعودية

المستخلص . هذه الورقة درست بيانات الأمطار والفيضانات الشهرية بغرض التعرف على تأثير تغير المقياس الزمني . تم تحليل بيانات وادي جيزان الشهرية الخاصة بالأمطار والسيول لمجالات التغير الزمني الخطي ، التعاقب ، الانحدار الذاتي والمتبقي العشوائي .

نُوقش التفسير الفيزيائي لهذه المكونات وقد وجد أن بعضها ذو قيمة في تحديد صيغ النماذج .