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ON THE EFFECT OF BED ROUGHNESS ON THE PROPERTIES OF THE HYDRAULIC JUMP

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ABSTRACT

The objective of this work is to investigate the effect of bed roughness on properties of the hydraulic jump. The characteristics of the jump are theoretically analysed using the relations of continuity, momentum, and energy for the one-dimensional flow. The significant effect of bed roughness is included in the present theoretical work. This provides expressions to obtain the jump properties such as the relative length of the jump and roller, the conjugate depth ratio, the loss of energy in the roller, the loss of energy in friction and the relative integrated bed shear force over rough beds. These expressions are also presented in their graphical form.

1. INTRODUCTION

The hydraulic jump is one of the most interesting and useful open-channel phenomena. This phenomenon has attracted the interest of a great number of scientists and engineers. To the scientists, the jump is still one of the mysteries of hydraulics, up till now they have not found a satisfactory analysis of its internal mechanism and its causes. To the engineers, the jump represents a useful tool to be used to mix chemicals, aerate water, and as an energy dissipator for spillways and outlet works so that objectionable scour in the downstream channel is prevented.

The hydraulic jump over smooth beds has received considerable attention in both the theoretical and experimental analysis. Very good agreement between theoretical and experimental results was obtained. However, relatively little information is available about the properties of the hydraulic jump over rough beds. This is due to the fact that less theoretical and experimental investigations have been made on this particular aspect of the phenomenon. Although the published work up to date on the hydraulic jump over rough bed is mainly experimental, the literature provides very little information on this subject. Leutheusser and Schiller [4] concluded that the whole question of the roughness effects on the hydraulic jump in open channel is still largely un[illegible]

2 THEORETICAL APPROACH

The theoretical study of the free hydraulic jump over rough bed requires the simplification of the hydraulic formulation of the problem. This is made with the following assumptions: the velocity distributions before and after the jump are uniform, the turbulence effect and the air entrainment are negligible, and the channel has a rectangular cross-section of sufficient width, and / or the bed roughness is so great relative to the side wall roughness to eliminate any appreciable side wall effect.

The present approach assumes a steady flow and adopts the following governing equations in the analysis :

(1) the law of conservation of mass (the continuity equation in one dimensional flow),

(2) the law of conservation of momentum (Newton's second law of motion in one dimensional flow), and

(3) the law of conservation of energy (Bernoulli's equation in one dimensional flow).

The definition sketch of the hydraulic jump in a prismatic horizontal rectangular channel is presented in Fig. 1. Sections 1 and 2 refer to the beginning and end of the jump is shown by the dashed lines in this figure.

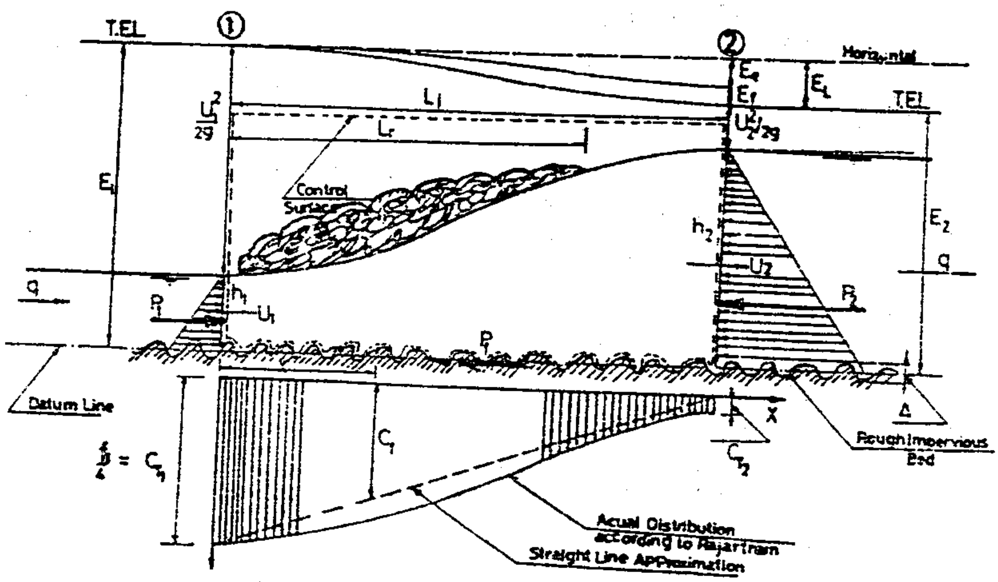


Fig. (1) Momentum and Energy Equations Applied to Hydraulic Jump in a Rough Horizontal Prismatic Rectangular Channel.

2.1 The Conjugate Depth Ratios over Rough Bed (h_2/h_1)

When Newton's Second law of motion is applied to the hydraulic jump the momentum equation can be derived. Hence, the summation of all external forces on a control volume of a liquid in any particular direction is equal to the change in momentum flux in that direction.

$$\sum Fx = \Delta(\rho\, q\, U) \qquad (1)$$

Referring to Fig. 1 and considering the control volume, Eq. 1 may be expressed as

$$P_1 - P_2 - P_f = \rho\, q\, (U_2 - U_1) \qquad (2)$$

where,
P_1 and P_2 are the pressure forces per unit width at sections 1 and 2 of the control volume respectively,
P_f is the bed integrated shear force per unit width over the jump length on the control volume adjacent to the channel bed,
ρ is the mass density of water,
U_1 is the average supercritical velocity at the beginning of the jump, and
U_2 is the average subcritical velocity at the end of the jump.

The values of P_1 and P_2 are calculated from the laws of hydrostatics, assuming linear pressure distribution at both ends of the hydraulic jump

$$P_1 = \frac{1}{2}\, \gamma\, h_1^2 \qquad (3)$$

$$P_2 = \frac{1}{2}\, \gamma\, h_2^2 \qquad (4)$$

where,
γ is the specific weight of water.

According to Rajaratnam [7] the boundary friction force per unit width, P_f may be obtained by the integration of the measured bed shear stress profile as follows :

$$P_f = \int_0^{L_j} \tau_o\, dx \qquad (5)$$

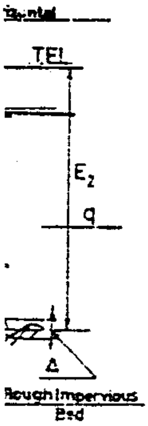
where,
τ_o is the local shear stress on the channel bed.

However, in the present study the value of τ_o is calculated from the following equation

$$\tau_o = C_f\, \frac{\rho\, U_1^2}{2} \qquad (6)$$

where, C_f is the local friction coefficient.

Substituting Eq. 6 into Eq. 5 we get,



$$P_f = \left(\frac{\rho U_1^2}{2}\right) \int_0^{L_j} C_f \, dx \tag{7}$$

In order to solve this integration a mathematical expression must be found for the variation of C_f along the jump length. The variation of C_f with the dimensionless distance along the channel bed x/h_2, where h_2 is the subcritical sequent depth of the jump is plotted by Rajaratnam [7] for the smooth beds and shown in Fig. 1. Assuming that the same trend of variation is also valid in rough channels and a straight line approximation is made to the experimental data with a maximum value of C_{f1} at the beginning of the jump and an approximately zero value of C_f ($C_{f2} \approx 0.0$) at the jump end. According to the above assumptions the variation of the local coefficient of friction C_f with the distance x may be expressed by the relation

$$C_f = C_{f1} \left(1 - \frac{x}{L_j}\right) \tag{8}$$

in which, C_{f1} is the skin friction coefficient at the jump toe.

The substitution from Eq. 8 into Eq. 7 and upon integration, we have,

$$P_f = \frac{C_{f1}}{4} \rho U_1^2 L_j \tag{9}$$

Inserting Eq. 3, Eq. 4 and Eq. 9 into Eq. 2 we get,

$$\frac{1}{2}\gamma h_1^2 - \frac{1}{2}\gamma h_2^2 - \frac{C_{f1}}{4} \rho U_1^2 L_j = \rho q (U_2 - U_1) \tag{10}$$

The simplification of Eq. 10 leads to the following

$$\left(\frac{h_2}{h_1}\right)^3 - \left[1 + 2F_1^2 - \frac{f_1}{8} F_1^2 \left(\frac{L_j}{h_1}\right)\right]\left(\frac{h_2}{h_1}\right) + 2F_1^2 = 0 \tag{11}$$

where, f_1 = the Darcy-Weisbach resistance coefficient at the jump toe, and

F_1 = the upstream Froude Number.

Eq. 11 may be conveniently written as

$$J^3 - A J + B = 0 \tag{12}$$

where,

$$J = h_2/h_1, \tag{13}$$

$$A = 1 + 2F_1^2 - \frac{f_1}{8} F_1^2 \left(\frac{L_j}{h_1}\right), \tag{14}$$

and

$$B = 2 F_1^2 \tag{15}$$

It is to be noted here that for f_1=0 Eq. 11 becomes the well-known Bèlanger equation for the free jump over the smooth bed given by :

$$\frac{h_2}{h_1} = \frac{1}{2}\left[\sqrt{1 + 8F_1^2} - 1\right] \tag{16}$$

Eq. 12 is a simple cubic equation and therefore, it has three roots. It is expected that at least two real roots exist representing supercritical and subcritical depths, but no complex roots are possible as they may appear in pairs ref.[2]. This will lead to the following result,

$$(B^2/4) - (A^3/27) \leq 0 \qquad (17)$$

In case where $(B^2/4) - (A^3/27) < 0$, three unequal roots may found in the form,

$$J = 2\sqrt{\frac{A}{3}}\ \cos\left[\frac{\Phi + 2n\Pi}{3}\right] \qquad (18)$$

where, n = 0, 1, and 2

and
$$\Phi = \cos^{-1}\left[\frac{-B}{2\sqrt{A^3/27}}\right] \qquad (19)$$

The first root (where n=0) is a realistic solution, representing the existence of a subcritical conjugate depth at section (2) of Fig. 1 .

The second root (where n=1) gives a negative value of the conjugate depth. This solution is physically impossible, and therefore can be discounted.

The third root (when n=2) represents the case where the hydraulic jump does not form in the control volume, and instead it will occur downstream of section 2, and hence the flow within the control volume is entirely supercritical.

Therefore the real solution for the conjugate depth ratio is given by

$$J = 2\sqrt{\frac{A}{3}}\ \cos\left[\frac{\Phi}{3}\right] \qquad (20)$$

Eq. 20 may be written in the functional form

$$J = \Psi\ (f_1,\ F_1,\ L_j/h_1) \qquad (21)$$

In order to calculate the values of the conjugate depth ratio J, at different values of the inlet Froude number F_1 and the average resistance coefficient (Darcy-Weisbach) at the jump inlet f_1, the relative jump length L_j/h_1 must be known. Therefore, another relation for the relative jump length is required to enable the determination of the conjugate depth ratio J.

2.2 Graphical representation of Possible Solutions of General Equation of The Hydraulic Jump over Rough Beds

The solutions of general equation of the hydraulic jump over rough beds (Eq. 12) is given by the relation 18. Neglecting the negative root, the conjugate depth ratio J in Eq. 18 is calculated with the aid of a digital computer and the results are plotted in Fig. 2 against the upstream Froude number F_1 for both the subcritical and the supercritical flow at the end of the control volume (section 2). These curves are plotted for different values of the term $D = f_1\ L_j/h_1$, between 0 and 10.

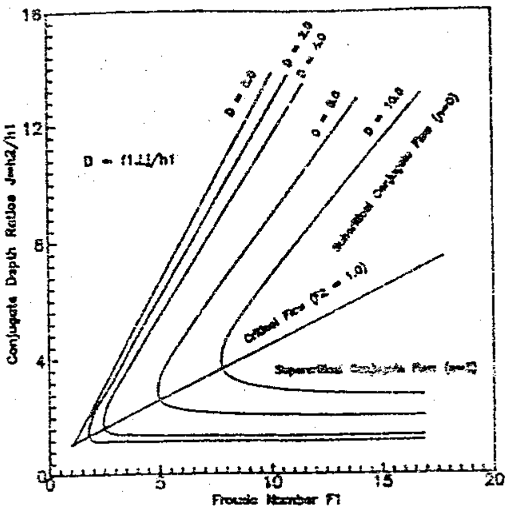


Fig. (2) Graphical Representation of The Possible Solutions for The General Jump Equation over Rough Beds

The upper part of the parabolic curves represent the subcritical conjugate depth while the lower part of the curves represent a nonexistent supercritical conjugate depth at section 2. The vertex of the curves are the theoretical limit which indicate that the tailwater is critical. It is to be noted that the effect of the bed roughness (D>0) causes a decrease in the conjugate depth ratio J below its corresponding value for the hydraulic jump over a smooth bed (D=0).

2.3 The Relative Length of Jump over Rough Bed (L_j/h_1)

A description of the hydraulic jump remains incomplete without mentioning its length. In 1976, Mehrotra published a paper about the length of the hydraulic jump [5], in which he claimed that he made the first attempt for an analytical treatment of the problem. His theoretical expression for the length of roller was based on the assumption that the roller accounts for all the loss of energy in the jump.

In the present study, the assumption that all the loss of energy in the hydraulic jump is due to roller only would be incorrect, because the rough bed frictional resistance cannot be ignored.

Therefore the loss of energy in the jump is assumed to be divided into two components, the first one is the loss of energy in the turbulent roller which comprises energy containing eddies according to Mehrotra [5].

The second is the energy dissipated by the friction on the channel bed. Both components of energy loss in the hydraulic jump are shown in Fig. 1.

Consequently the total amount of energy dissipated per second in a hydraulic jump of unit width occurring over a rough bed may be expressed by the following

$$\gamma\ q\ E_L = \gamma\ q\ E_r + \gamma\ q\ E_f \qquad (22)$$

where,

$\gamma\ q\ E_L$ = the total amount of energy dissipation by the jump,

$\gamma\ q\ E_r$ = the loss of energy in turbulent roller, and

$\gamma\ q\ E_f$ = the loss of energy by friction on the channel bed.

The determination of each term in the right hand side of equation 22 is presented separately in the following :

1) The Loss of Energy in The Turbulent Roller:

The energy flux lost in the roller, according to Mehrotra [5] was given by :

$$\gamma\ q\ E_r = \gamma\ q\ F_1^2 h_1 \frac{1}{\sqrt{k_j}} \frac{(J+1)^3}{J^3} \sqrt{\left(\frac{L_j}{h_1}\right)^2 + (J-1)^2} \qquad (23)$$

where, k_j = dimensionless constant for the jump length which is equal to 35,000 [5].

2) The Loss of Energy due to The Friction on The Channel Bed:

The jump lost energy due to the friction on the bed may be calculated from the work done per second to overcome the friction force on the rough bed. Thus, we have

$$\gamma\ q\ E_f = \int_0^{L_j} U\ dP_f \qquad (24)$$

where, dP_f is the integrated bed shear force along the distance dx of the channel bed as shown in Fig. 3. Hence,

$$dP_f = \tau_o\ dx \qquad (25)$$

Inserting this value into Eq. 26 we obtain

$$\gamma\ q\ E_f = \int_0^{L_j} U\ \tau_o\ dx \qquad (26)$$

also, inserting Eq. 6 into Eq. 26 we have

$$\gamma\ q\ E_f = \frac{\rho\ U_1^2}{2} \int_0^{L_j} C_f\ U\ dx \qquad (27)$$

The variation of the maximum velocity along the jump length was measured by Rajaratnam [7]. Approximate linear relation is assumed as shown in Fig. 4.

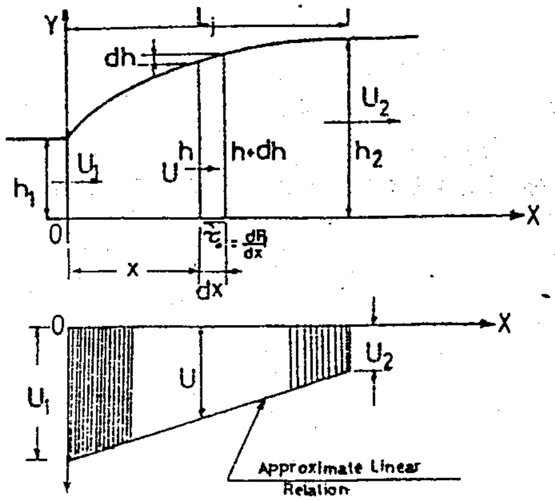


Fig. (3) Definition Sketch of The Model Used in The Analysis.

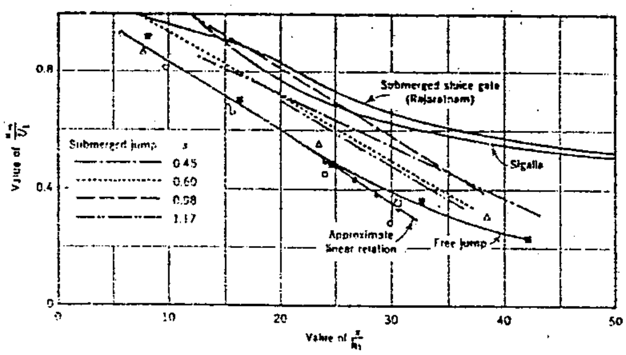


Fig. (4) Variation of u_m / U_1 with x/h_1.

Assuming that this relation is also valid for the mean velocity in rough channels. Referring to Fig. 3, one may deduce the relation for the variation of the mean velocity along the length of the jump

$$U = U_1 - (U_1 - U_2)\left(\frac{x}{L_j}\right) \qquad (28)$$

Inserting both Eq. 5 and Eq. 28 into Eq. 27 we obtain

$$\gamma\, q\, E_r = \frac{\rho U_1^2}{2} \int_0^{L_j} C_{f_1}\left(1 - \frac{x}{L_j}\right)\left[U_1 - (U_1 - U_2)\left(\frac{x}{L_j}\right)\right] dx \qquad (29)$$

Eq. 29 may be written as

$$\gamma\, q\, E_f = \gamma\, q\, h_1 \frac{f_1}{48} F_1^2 \left(\frac{L_j}{h_1}\right) \left[2 + \frac{1}{J} \right] \tag{30}$$

3) *The Total Loss of Energy in The Hydraulic Jump:*

The total energy dissipated in the jump can be calculated from the difference of energy of the flow at the jump upstream and downstream sections. Hence, applying Bernoulli's equation between sections (1) and (2) of the hydraulic jump (Fig. 1), we get

$$\gamma\, q\, E_L = \gamma\, q\, (E_1 - E_2) \tag{31}$$

where, $\gamma\, q\, E_L$ is the total energy loss per second,

E_1 is the specific energy at the jump upstream section, and

E_2 is the specific energy at the jump downstream section.

Eq. 31 may be expressed in terms of Froude Number F_1 and the conjugate depth ratio J as,

$$\gamma\, q\, E_L = \gamma\, q\, h_1 \left\{ 1 - J + \frac{1}{2} F_1^2 \left[1 - \frac{1}{J^2} \right] \right\} \tag{32}$$

Also, inserting Eqs. 23, 30 and 32 into Eq. 22 and simplifying. The relation may put in the general form as,

$$k_{r,j} \left\{ \left(\frac{f_1}{48}\right)^2 F_1^4 J^6 \left[2 + \frac{1}{J} \right]^2 - F_1^4 \frac{1}{k_{r,j}} (J+1)^6 \right\} \left(\frac{L_{r,j}}{h_1}\right)^2$$

$$- k_{r,j} \left\{ \frac{f_1}{24} \left[J^3 (1-J) + \frac{1}{2} F_1^2 J (J^2 - 1) \right] F_1^2 J^3 \left[2 + \frac{1}{J} \right] \right\} \left(\frac{L_{r,j}}{h_1}\right)$$

$$+ \left\{ k_{r,j} \left[J^3 (1-J) + \frac{1}{2} F_1^2 J (J^2 - 1) \right]^2 - F_1^4 (J+1)^6 (J-1)^2 \right\}$$

$$= 0 \tag{33}$$

in which, $k_{r,j}$ are constants in Eq. 33 only one subscript (r or j) is to be taken at a time in the right order.

$L_{r,j}$ may be the length of roller or the length of jump. Eq. 33 can be written in the following form :

$$a\, X^2 - b\, X + c = 0 \tag{34}$$

where,

$$a = k_{r,j} \left\{ \left(\frac{f_1}{48}\right)^2 F_1^4 J^6 \left[2 + \frac{1}{J} \right]^2 - F_1^4 \frac{1}{k_{r,j}} (J+1)^6 \right\} \tag{35}$$

$$b = k_{r,j} \left\{ \frac{f_1}{24} \left[J^3 (1-J) + \frac{1}{2} F_1^2 J (J^2 - 1) \right] F_1^2 J^3 \left[2 + \frac{1}{J} \right] \right\}$$

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$$c = \left\{ k_{r,j} \left[J^{3}(1-J) + \frac{1}{2} F_1^2 J (J^2 - 1) \right]^2 - F_1^4 (J+1)^{6} (J-1)^2 \right\} \qquad (37)$$

and $X = \left(\frac{L_{r,j}}{h_1}\right)$ (38)

Eq. 34 is a quadratic equation and therefore has two roots. The solution of Eq. 34 is in the form :

$$\frac{L_{r,j}}{h_1} = \frac{b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (39)$$

It is to be noted here that one of the two roots of Eq. 34 is the real solution (when the negative sign of Eq. 39 is considered). The other solution would be physically impossible. Thus the real solution of Eq. 34 is given by

$$\frac{L_{r,j}}{h_1} = \frac{b - \sqrt{b^2 - 4ac}}{2a} \qquad (40)$$

Dividing both sides of Eq. 40 by h_2/h_1, we get the expression for $L_{r,j}$ developed by Mehrotra [5] for the smooth bed case ($f_1=0$),

$$\frac{L_{r,j}}{h_2} = \left\{ \frac{k_{r,j}\left(\frac{h_1}{h_2}\right)^2 \left[1 - \frac{h_2}{h_1} + \frac{F_1^2}{2}\left[1 - \left(\frac{h_1}{h_2}\right)^2\right]\right]^2}{F_1^4 \left(1 + \frac{h_1}{h_2}\right)^6} - \left(\frac{h_1}{h_2}\right)^2 \left(\frac{h_2}{h_1} - 1\right)^2 \right\}^{1/2} \qquad (41)$$

Eq. 40 may be written in the functional form

$$\frac{L_{r,j}}{h_1} = \theta\,(f_1, F_1, J, k_{r,j}) \qquad (42)$$

Inserting Eq. 42 into Eq. 21 we get the functional form

$$J = \omega\,(f_1, F_1, J, k_j) \qquad (43)$$

Eq. 43 is an implicit function of J which may be solved for given values of f_1 and F_1 by iterative procedure. Consequently, the solution may be substituted into Eq. 40 to give the values of $L_{r,j}/h_1$. Then other characteristics of the hydraulic jump may be determined.

2.4 The Relative Integrated Bed Shear Force (P_f/P_1)

The relative integrated bed shear force is to be deduced for hydraulic jumps occurring in horizontal prismatic rectangular channels with a rough bed. The momentum equation between the toe and the heel of the jump may be given by

$$\frac{P_f}{\gamma} = M_1 - M_2 \qquad (44)$$

where,

M = the momentum function given by $(q^2/gh + h^2/2)$,

P_f = the integrated bed shear force,

1 and 2 are the suffix corresponding to sections at toe and heel of the jump respectively, and

γ = the specific weight of water.

Eq. 44 may be put in the form

$$\frac{P_f}{\gamma} = \left(\frac{q^2}{gh_1} + \frac{h_1^2}{2}\right) - \left(\frac{q^2}{gh_2} + \frac{h_2^2}{2}\right) \qquad (45)$$

Simplifying Eq. 45 we get

$$P_f/P_1 = (J-1)\left[\frac{2F_1^2}{J} - (1+J)\right] \qquad (46)$$

where,

P_1= the hydrostatic pressure force at the jump toe which is given by $(0.5\ \gamma h_1^2)$.

For the case of smooth bed (when P_f=0), Eq. 46 reduces to

$$J = 0.5\left(\sqrt{1 + 8F_1^2} - 1\right) \qquad (47)$$

which is the familiar equation of a hydraulic jump in rectangular channel derived by Bèlanger.

2.5 The Total Relative Energy Loss

The relative energy dissipated in the hydraulic jump in a horizontal rectangular channel E_L/h_1 may be expressed as,

$$E_L/h_1 = \left\{1 - J + \frac{1}{2} F_1^2\left[1 - \frac{1}{J^2}\right]\right\} \qquad (48)$$

The relative specific energy at the inlet section of the jump may be expressed as,

$$E_1/h_1 = \frac{1}{2}\left(2 + F_1^2\right) \qquad (49)$$

Dividing Eq. 48 by Eq. 49 and rearranging we get the relative energy loss E_L/E_1,

$$E_L/E_1 = 1 - \frac{1}{J^2}\left[\frac{2J^3 + F_1^2}{2 + F_1^2}\right] \qquad (50)$$

2.6 The Turbulent Roller Relative Energy Loss

Dividing Eq. 23 by $\gamma q h_1$ we obtain

$$E_e/h_1 = F_1^2\ \frac{1}{\sqrt{k_j}}\ \frac{(J+1)^3}{J^3}\sqrt{\left(\frac{L_j}{h_1}\right)^2 + (J-1)^2} \qquad (51)$$

2.7 The Bed Friction Relative Energy Loss

Dividing Eq.(30) by $\gamma q h_1$ we obtain,

$$E_f/h_1 = \frac{f_1}{48} F_1^2\left(\frac{L_j}{h_1}\right)\left[2 + \frac{1}{J}\right] \qquad (52)$$

2.8 Efficiency of Jump

The ratio of the specific energy after the jump to that before the jump is defined as the efficiency of the jump. Hence,

$$\eta = E_2/E_1 = 1 - E_L/E_1 \tag{53}$$

Inserting Eq. 50 into Eq. 53 we get the following expression for the hydraulic jump efficiency

$$\eta = E_2/E_1 = \frac{1}{J^2}\left[\frac{2\,J^3 + F_1^2}{2 + F_1^2}\right] \tag{54}$$

2.9 Solution of The General Equation of The Hydraulic Jump over Rough Beds Using Iterative Procedure

The general equation of the hydraulic jump over rough beds (Eq. 43) is to be solved for different values of the coefficient of friction f_1 using iterative procedure with the aid of digital computer. All iterative techniques used for solving algebraic equations start with an initial estimate for one of the unknown parameters. In this case, this initial estimate is made for the value of conjugate depth ratio over smooth bed for a given value of Froude number F_1. This initial estimate is improved through a sequence of operations having a finite number of steps. The improvement is achieved by the new estimate in a repetition of the same sequence of operations. The cycle is repeated until the desired specified accuracy is attained. The degree of accuracy may be ensured by calculating enough cycles.

3. Graphical Representation of the Theoretical Solution

Eq. 20 and Eq. 40 are solved with the aid of digital computer and the resulting conjugate depth ratios J, relative jump and roller lengths $L_{r,j}/h_1$ are shown in Figs. 5 and 6 respectively. These curves are computed for the values of coefficient of friction f_1 of 0.0, 0.02, 0.1, and 0.5, and Froude Number F_1 varying from the value of 1.5 to 15. All jump properties derived in sec. 2 are computed for the above mentioned range of f_1 and F_1, and are graphically represented in the following figures.

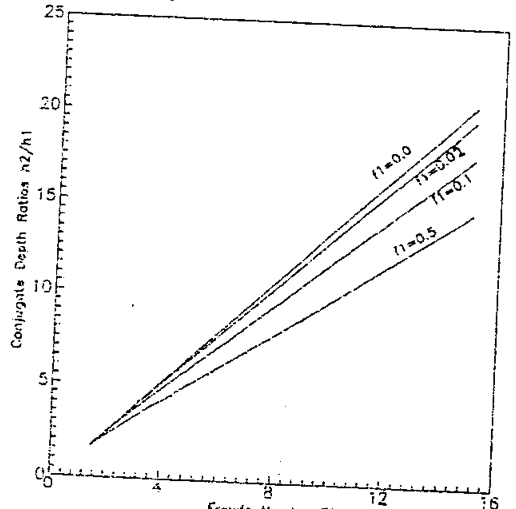

Fig. (5) Theoretical Relation for The Conjugate Depth Ratios h2/h1 as a Function of F1 and f1.

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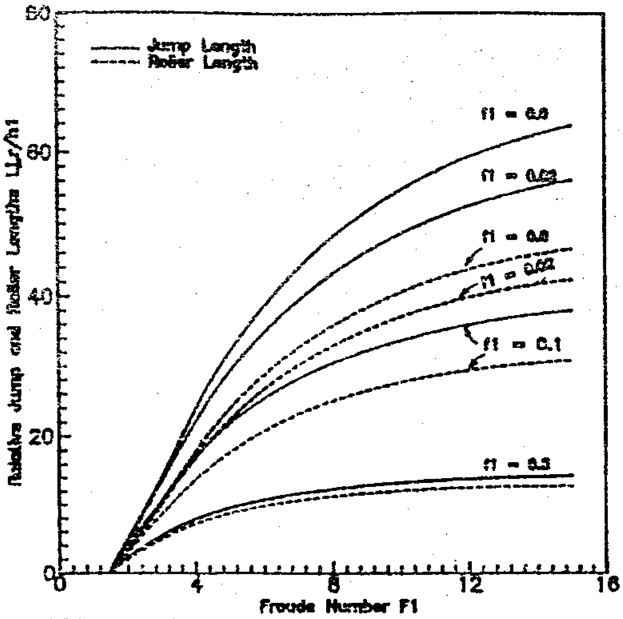

Fig. (6) Theoretical Relation for The Relative Lengths of Jump and Roller Lj,r/h1 as a Function of F1 and f1

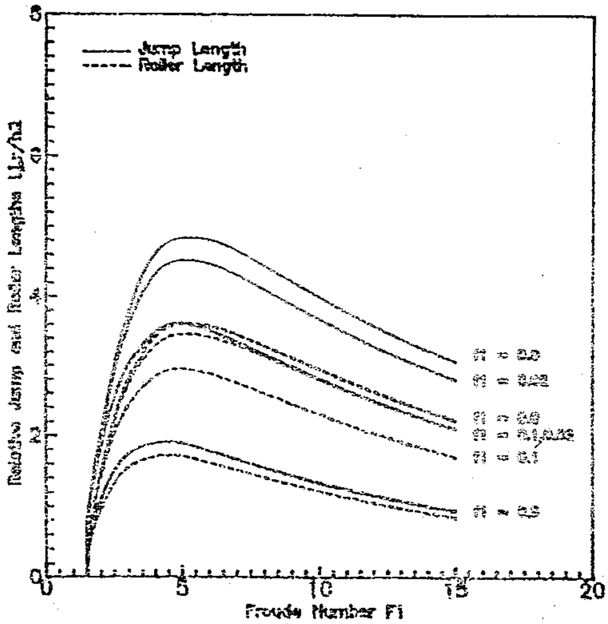

Fig. (7) Theoretical Relation for The Relative Lengths of Jump and Roller Lj,r/h2 as a Function of F1 and f1

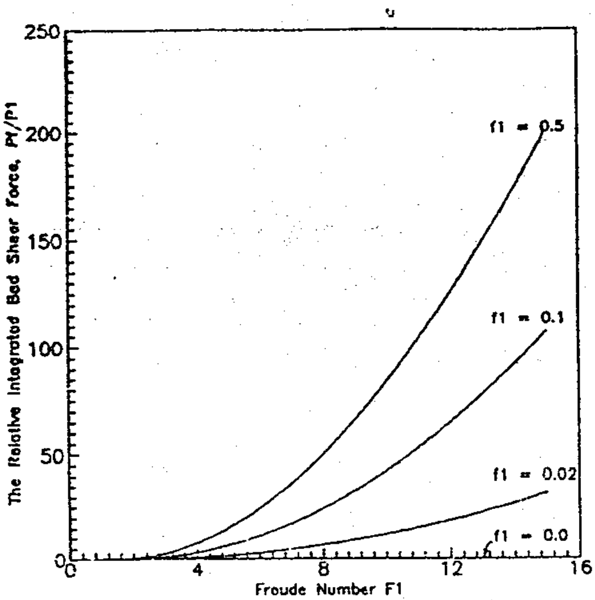

Fig. (8) Theoretical Relation for The Relative Integrated Bed Shear Force Pf/P1 as a Function of F1 and f1.

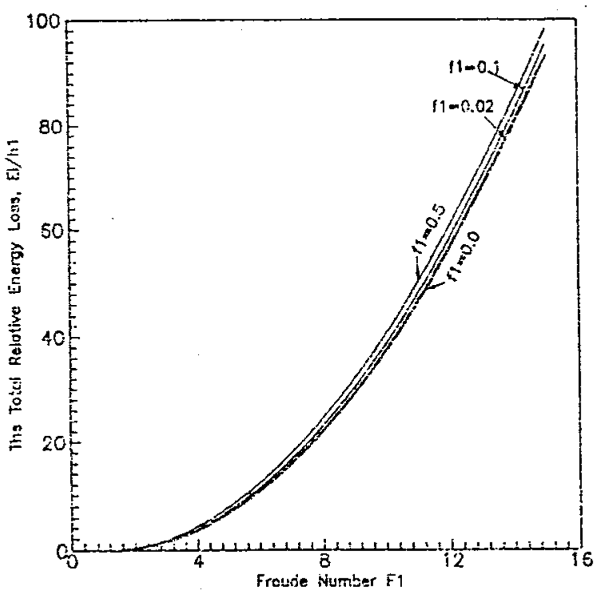

Fig. (9) Theoretical Relation for The Relative Total Energy Loss EL/h1 as a Function of F1 and f1

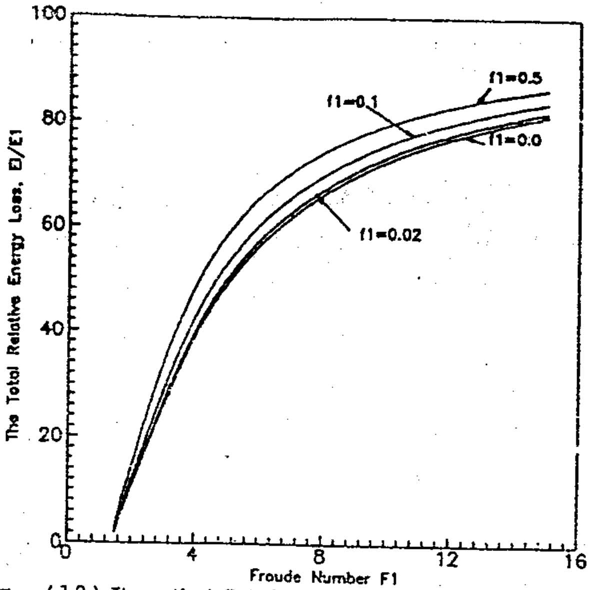

Fig. (10) Theoretical Relation for The Relative Total Energy Loss El/E1 as a Function of F1 and f1

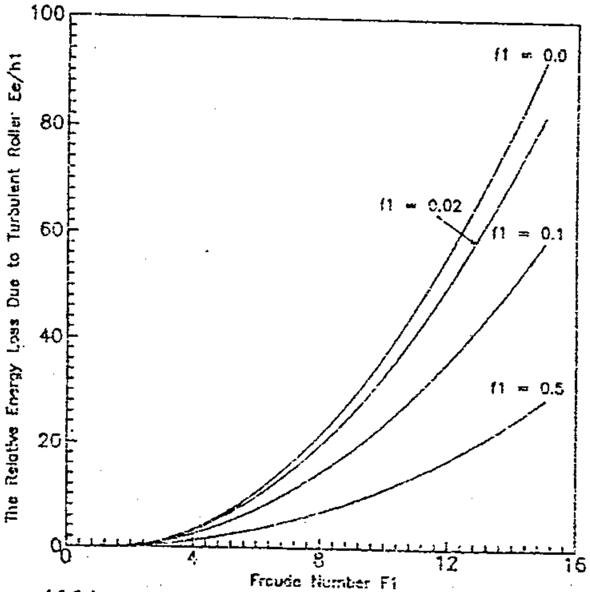

Fig. (11) Theoretical Relation for The Relative Energy Loss Due to Turbulent Roller Ee/h1 as a Function of F1 and f1.

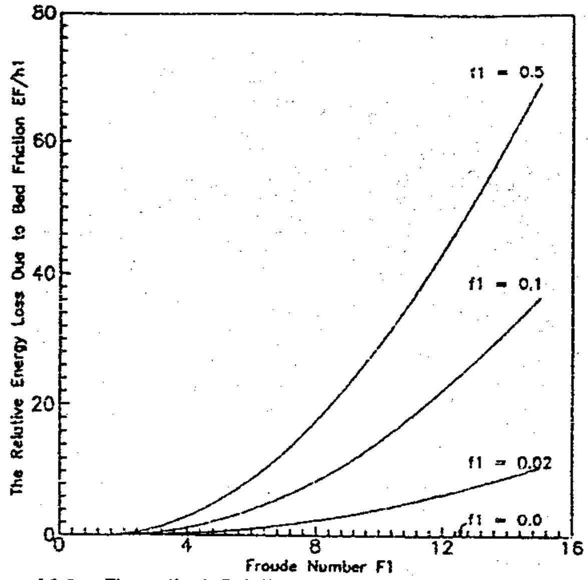

Fig. (12) Theoretical Relation for The Relative Energy Loss Due to Bed Friction Ef/h1 as a Function of F1 and f1

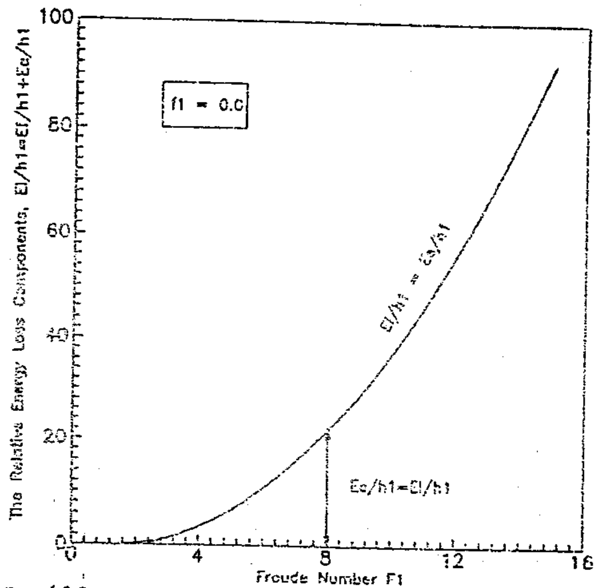

Fig. (13) Theoretical Relation for The Relative Energy Loss Components for f1 = 0.0

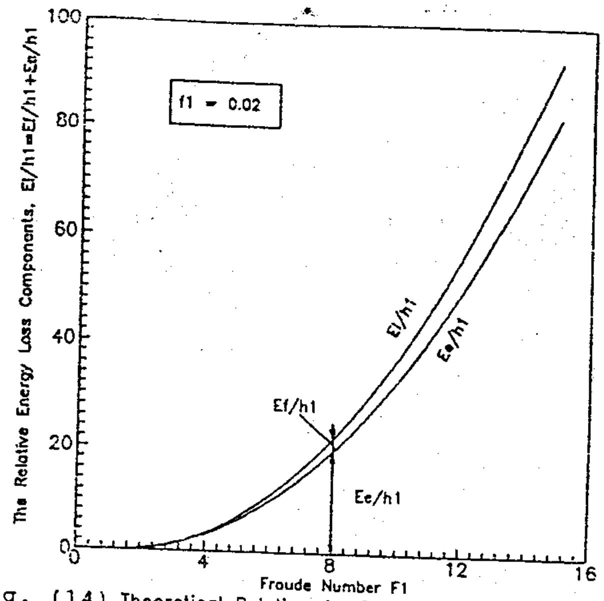

Fig. (14) Theoretical Relation for The Relative Energy Loss Components for f1 = 0.02

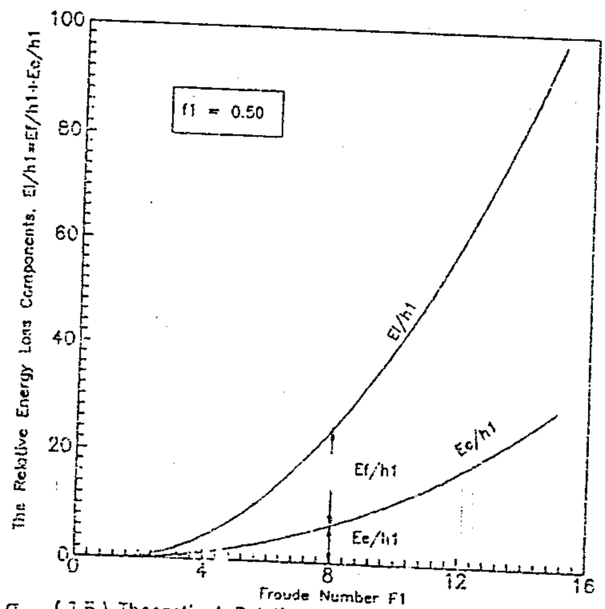

Fig. (15) Theoretical Relation for The Relative Energy Loss Components for f1 = 0.5

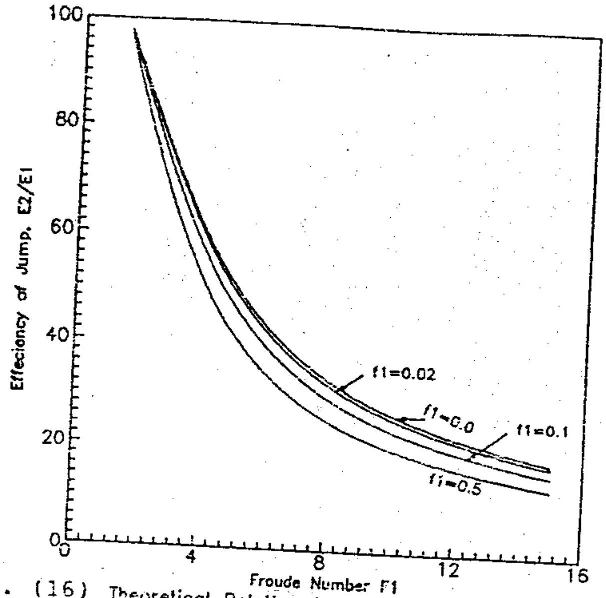

Fig. (16) Theoretical Relation for The Effeciency of Jump E2/E1 as a Function of F1 and f1

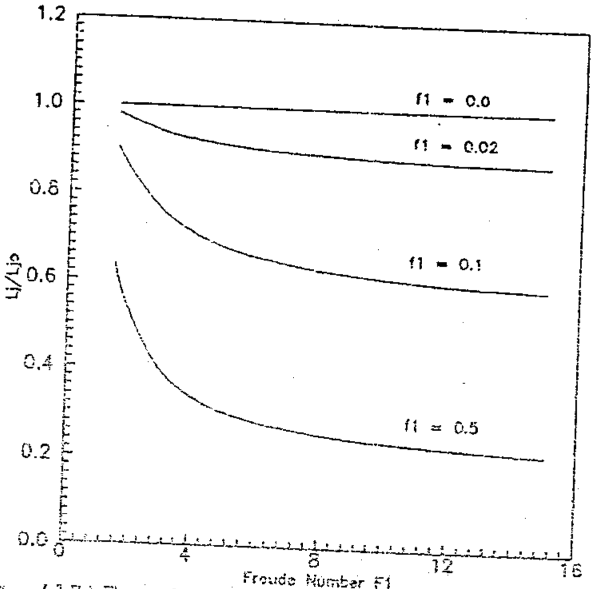

Fig. (17) Theoretical Relation for The Ratio of Jump Length over Rough Bed to Jump Length over Smooth Bed as a Function of F1 and f1

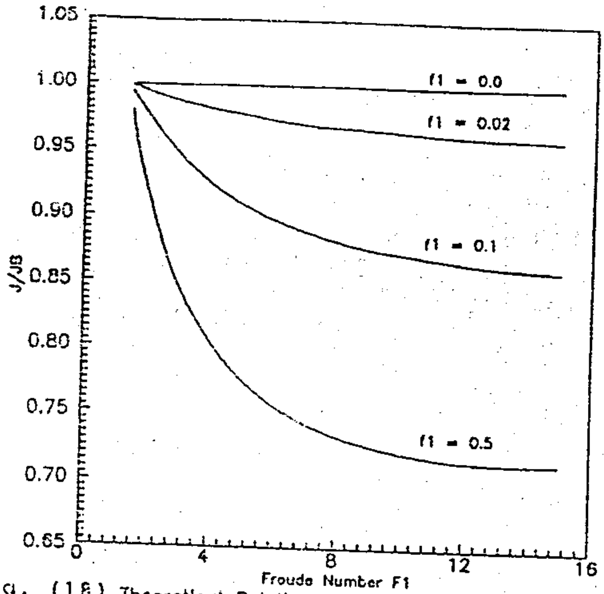

Fig. (18) Theoretical Relation for The Ratio of Conjugate Depth Ratio over Rough Bed to That over Smooth Bed as a Function of F1 and f1

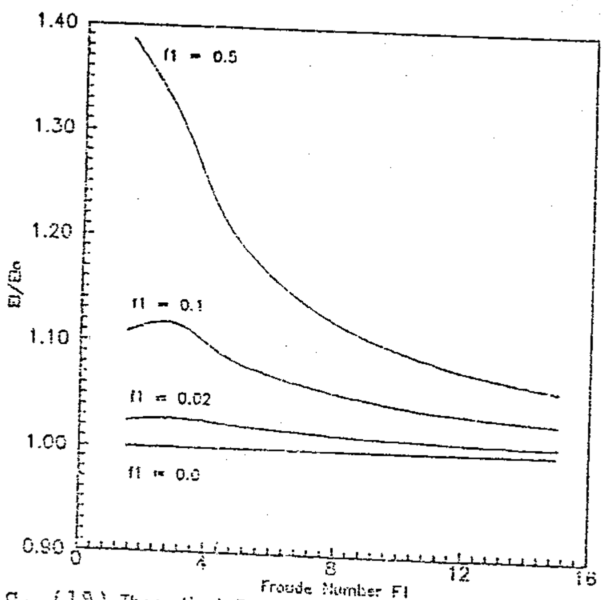

Fig. (19) Theoretical Relation for The Energy Loss over Rough Bed to That over Smooth Bed as a Function of F1 and f1.

4. Analysis and Discussions of the Theoretical Results

4.1 The Conjugate Depth Ratios

Eq. 20 is represented graphically in Fig. 5. This equation shows that with the increase of the value of the coefficient of friction there is a tendency towards a corresponding decrease in the conjugate depth ratios for the same value of Froude number. For the frictionless bed case (f_1=0.0) Eq. 20 leads to the classical hydraulic jump Eq. 16 which is represented by the upper line of Fig. 5 . One may also conclude that the effect of bed friction results in decrease of the tailwater depth.

4.2 The Relative Lengths of Jump and Roller

The common dimensionless parameters used for describing the jump and roller lengths are $L_{j,r}/h_1$, $L_{j,r}/h_2$, and $L_{j,r}/(h_2-h_1)$.

The plot of Eq. 40 which represent the variation of $L_{j,r}/h_1$ versus the Froude number F_1 for different values of the coefficient of friction f_1 is shown in Fig. 6. This figure shows that the increasing of bed friction results in the decreasing of both the jump length and the roller length. It is also to be noted here that at high values of friction factors the roller length gets closer to the jump length. When the value of f_1=0.0 Eq. 40 gives a similar form to the equation of Mehrotra used for smooth bed rectangular channels.

Another plot for the relative jump and roller length is shown in Fig. 7. This plot is desirable, because the resulting curve displays a relatively flat portion for the range of well established jumps. Fig. 7 gives the same conclusions for variation of $L_{j,r}/h_2$ versus F_1, which was previously made from Fig. 6. The value of f_1=0.0 in Fig. 7 gives the graphical representation of Eq. 41 which was derived by Mehrotra.

4.3 The Relative Integrated Bed Shear Force

Eq. 46, which represents the ratio of the integrated bed shear force to the hydrostatic force at the beginning of the jump is plotted in Fig. 8 at different values of the friction coefficient. This equation shows that the integrated bed shear force increases with the respective increase of the friction coefficient at constant Froude number F_1. In the case of a smooth bed channels (f_1=0.0), the relative integrated bed shear force is zero and this can be represented by the X-axis. The increasing of the integrated bed shear force accounts for the resulting decrease in the conjugate depth ratio, shown in Fig. 5.

4.4 The Total Relative Energy Loss

Eqs. 48 and 50 are represented graphically by the Figs. 9 and 10 respectively. Fig. 9 represents the relation between the total relative energy loss E_L/h_1 and the upstream Froude number F_1 for different values of the coefficient of friction f_1.

This figure shows a slight increase in the total energy loss of the hydraulic jump occurring over rough beds compared with the respective total energy loss on smooth beds. Fig. 10 displays the relation between the total relative energy loss E_L/E_1 as a function of both Froude number F_1, and the coefficient of friction f_1.

4.5 The Components of The Relative Energy Loss

Eqs. 51 and 52 are plotted in Figs. 11 and 12 respectively. Eq. 51 represents the relative energy loss due to turbulent roller E_o/h_1 as a function of the Froude number F_1 and the coefficient of friction f_1. It is clear from Fig. 11 that with the increasing of the coefficient of friction, the relative energy loss due to turbulent roller decreases with the decrease in the roller length and consequently in the total length of the hydraulic jump. In case when the bed is smooth (f_1=0.0) the total energy loss is only due to turbulent roller. This case is represented by the upper curve in Fig. 11.

Eq. 52 represents the relative energy loss due to bed friction E_f/h_1 as a function of the Froude number F_1 and the coefficient of friction f_1. The curves in Fig. 12 show the increase of relative energy loss due to bed friction with the increase of the friction coefficient. For the assumed smooth bed case (f_1=0.0) there is no energy loss due to friction, this is represented by X-axis of Fig. 12 which indicates that the loss of energy is only due to turbulent roller.

Fig. 13, Fig. 14 and Fig. 15 show the relative energy loss components at the different values of the friction coefficient of 0.0, 0.02, and 0.5 respectively.

Fig. 13 indicates that the energy loss in the jump is due to turbulent roller only. Thus, the curve for the energy loss due to turbulent roller E_o/h_1 coincides with the curve of total energy loss E_L/h_1. This represents the case of a classical hydraulic jump (free jump on smooth wide rectangular channel).

Fig. 14 shows that for f_1=0.02 a small part of energy loss is due to bed friction and the remaining part is due to turbulent roller which is greater than the loss due to bed friction.

Fig. 15 shows that for f_1= 0.5 a large part of energy loss is due to bed friction and the remaining part is due to turbulent roller which is less than the loss due to bed friction. From the above discussion we may conclude that the increase of bed friction causes the decrease of the energy loss due to turbulent roller. This means that the produced jump will be shorter and more stable. This also, reduces the amount of wave formation in the jump.

4.6 The Efficiency of Jump

Eq. 54 gives a theoretical relation for the efficiency of hydraulic jump as a function of Froude number F_1 and coefficient of friction f_1.

This equation is represented graphically in Fig. 16 which shows a decrease in the efficiency of the jump with the increase of the value of the friction factor, Thus the exit energy of the jump will be less than the corresponding exit energy for the smooth bed case. This conclusion is important in the design of the hydraulic jump as an energy dissipator, to protect channels from serious scour.

4.7 Comparison between Hydraulic Jumps Occurring on Rough and Smooth Beds

The present theoretical study also enables the comparison of the parameters of the hydraulic jumps occurring on rough and smooth beds. These parameters are the jump length, the conjugate depth ratio and the energy loss.

Fig. 17 shows the ratio of jump length over a rough bed to jump length over smooth bed L_j/L_{jo} as a function of Froude number F_1 and the coefficient of friction f_1. This ratio decreases with the Froude number F_1. However, this decrease in the ratio of the length of jumps formed over the rough and the smooth beds is only significant for high values of the friction factor. For example at $F_1 = 8$, we read $L_j/L_{jo}=0.95$ for $f_1=0.02$, $L_j/L_{jo}=0.62$ for $f_1=0.1$, and $L_j/L_{jo}=0.31$ for $f_1=0.5$. Fig 17 also shows that at high values of Froude number $F_1>8$ the length ratio remains approximately unchanged for the different values of the friction factor. Therefore the length ratio of the jumps is dependent only upon the coefficient of friction for the high Froude number (F_1 greater than about 8).

Fig. 18 shows the ratio of the conjugate depth ratio over a rough bed to the corresponding conjugate depth ratio over a smooth bed as a function of the Froude number F_1 and the coefficient of friction f_1. A reduction in the conjugate depth ratio of jumps occurring over rough beds compared with jumps formed over smooth beds is observed. Therefore, a reduction in the tailwater depth is observed for jumps over a rough bed. This is only significant at Froude number greater than about 4. Also, it is to be noted that for Froude number $F_1>8$ this ratio remains approximately unchanged. Furthermore, in this range of Froude number the friction factor causes the decreasing of the tailwater depth and consequently the value of the conjugate depth ratio.

Fig. 19 shows the variation of the energy loss ratio of the hydraulic jumps occurring over rough and smooth beds. The curves displays a decreasing trend of the energy loss ratio with the increasing of Froude numbers in the range from about 3 to 15. Also, a slight increase in the energy loss ratio is observed for the case $f_1 = 0.02$ and 0.1 in the Froude number range 1.5 to 3.0 which represent undular and weak jumps.

5. Conclusions

1. For free hydraulic jump occurring on rough beds, the momentum equation which neglects the total bed shear force can not satisfactorily predict the jump conjugate depth.

A theoretical expression including the total bed shear force has been developed to predict the conjugate depth ratios h_2/h_1 as a function of the friction factor at the jump toe f_1, Froude number F_1 and the relative jump length L_j/h_1, (Eq. 20).

2. The theoretical equation for the relative jump length over a smooth bed derived by Mehrotra [5] is modified to represent the hydraulic jump formed on rough beds. Therefore, a theoretical equation is obtained for the relative jump length L_j/h_1 as a function of Froude number F_1 conjugate depth ratios h_2/h_1 and the friction factor at the jump toe f_1, (Eq. 40).

3. A theoretical equation for predicting the energy loss due to bed friction is derived from the principle of work done of the friction force acting on the rough bed of the hydraulic jump, (Eq. 30).

4. For the hydraulic jump occurring on a smooth bed, the length of jump is of no consequence in the determining of the jump parameters, whereas in rough bed jumps their lengths would play a significant role.

5. The conjugate depth ratio (h_2/h_1) over a rough bed is significantly lower than the corresponding depth ratio over the smooth bed for the flows of Froude number $F_1 \geq 8.0$ (Fig. 5 shows that for $F_1 = 12$ the value of $h_2/h_1 = 16.5$, 15.9, 14.3, and 11.8 for $f_1 = 0.0$, 0.02, 0.1, and 0.5 respectively). Consequently the tailwater depth of the latter would be higher than tailwater level on the rough bed.

6. The jump length over a rough bed is shorter than the length of the corresponding jump over a smooth bed (Fig. 17 shows that for $F_1 = 8$, the value of $L_j/L_{jo} = 0.95$ for $f_1 = 0.02$, $L_j/L_{jo} = 0.62$ for $f_1 = 0.1$, and $L_j/L_{jo} = 0.31$ for $f_1 = 0.5$).

7. The relative energy loss, E_L/E_1 of the jump over a rough bed is greater than the respective energy loss of the corresponding jump occurring on the smooth bed (Fig. 10 shows that for $F_1 = 9.0$, $E_L/E_1 = 69.9$, 70.8, 73.3, and 77.3 for $f_1 = 0.0$, 0.02, 0.1, and 0.5 respectively).

8. The rough floor stilling basin will therefore have a low exit kinetic energy when compared with stilling basin with relatively smooth bed.

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APPENDIX : NOTATIONS.

The following symbols are used in this paper,

Symbol	Definition
A	Parameter given by Eq. 14.
a	Parameter given by Eq. 35.
B	Parameter given by Eq. 15.
b	Parameter given by Eq. 36.
c	Parameter given by Eq. 37.
C_f	Local skin friction coefficient, $\tau_o/0.5\rho U_1^2$.
C_{f1}	Skin friction coefficient at jump toe.
C_{f2}	Skin friction coefficient at jump heel.
D	Parameter, $f_1 L_j/h_1$.
E_1	Specific energy at section 1-1.
E_2	Specific energy at section 2-2.
E_e	Energy loss due to turbulent roller.
E_f	Energy loss due to bed friction.
E_L	Total energy loss over any test bed.
E_{Lo}	Total energy loss over smooth test bed.
f_1	Darcy-Weisbach friction factor at jump toe.
F_1	Supercritical Froude number, $U_1/\sqrt{gh_1}$.
F_x	Force in x-direction acting on the control volume containing the jump, Fig. 1.
g	Acceleration due to gravity.
h_1	Effective depth of flow at the jump toe.
h_2	Effective depth of flow at the jump heel.
J	Conjugate depth ratios, h_2/h_1.
k_r	Constant for roller length
k_j	Constant for jump length,
L_j	Length of jump, Fig. 1.
L_r	Length of roller, Fig. 1.

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Symbol	Definition
L_{jo}	Length of jump over smooth test bed.
M	The momentum function given by $(q^2/gh + h^2/2)$
P_1	Pressure force per unit width at jump toe.
P_2	Pressure force per unit width at jump heel.
P_f	Integrated bed shear force per unit width
q	Discharge per unit width, Q/W.
U_1	Average velocity upstream of the jump.
U_2	Average velocity downstream of the jump.
W	Width of channel.
Z	Drop Heigh of the Weir Model.
γ	Specific weight of water.
Δ	Effective height of roughness elements,
η	Efficiency of hydraulic jump, E_2/E_1.
θ	An arbitrary function.
μ	Dynamic viscosity.
ν	Kinematic viscosity.
ρ	Density.
Σ	Summation.
τ_o	Local bed shear stress.
Φ	Parameter given by Eq. 19.
Ψ	An arbitrary function.
ω	An arbitrary function.