

# Transient response of an adaptive beam with embedded piezoelectric microactuators

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## ABSTRACT

The transient response and the damping of a cantilevered adaptive beam is explored in this paper. The adaptivity of the beam comes from embedded piezoelectric actuators which are used to enhance the damping of the beam. The dynamic model is based on Hamilton's variational principle and the interactions between the beam and the devices are modeled using Eshelby's equivalent inclusion method. The Rayleigh-Ritz technique is used to obtain the approximate behavior of the structure.

## 1. INTRODUCTION

A smart structure can be defined as a system which has the capability of sensing a stimulus and responding to it in an appropriate fashion. So, the concept of adaptive structures pre-supposes that the structure is capable of responding to a changing environment or external stimulus in a prescribed manner.

Piezoelectric materials have become increasingly important as sensors and actuators in active structural control applications. Sensors are used to sense the external stimuli and their output is fed to actuators to modify, tune and control the structural response. These devices are either embedded within the structure or bonded to the surface of the structure. Most of the bonded actuators are relatively large in order to achieve adequate authority. On the other hand, embedding devices throughout the volume of the host allows sufficiently high volume fractions of devices, without having to use excessively large devices. The advantage of using many small devices over few large ones has obvious implications in terms of reduced obtrusiveness and improved reliability. This paper presents an analysis technique for a system with many embedded micro-devices.

Several models of the mechanical interaction between piezoelectric devices and structures to which they are bonded or embedded are available in the literature. One of these models has been developed by Dasgupta and Sirkis [1992] to model fiber optic sensors, as embedded cylindrical devices, using displacement function methods. Other models include, one-dimensional eigen-function approximations [Crawley and de Luis, 1987], simple beam model [Crawley and Lazarus, 1991], and pin force model [Lin and Rogers, 1992]. Also, variational methods have been developed, including Rayleigh Ritz methods [Hagood et al, 1990], strain energy method [Wang and Rogers, 1991], and finite element methods [Robbins and Reddy, 1991]. Most of the available models in the literature are for devices of the same length scale as the surrounding host structure.

In this paper the devices are small compared to the characteristic dimensions of the host. Also, this paper presents the transient response of an adaptive beam using Eshelby's classical equivalent inclusion

method, for the first time. Equivalent inclusion techniques are used to model the mechanical interaction between the embedded micro-devices and the host structure. This model differs from what has been presented in previous work, [Alghamdi and Dasgupta, 1993a], in the way the boundary terms are handled. The modeled structure is a cantilever beam made of ALPLEX plastic containing four rows of PZT-5H piezoelectric micro-devices. The generalized dynamic equations of motion are developed with Hamilton's Principle and then solved by combining Eshelby's eigenstrain method with Rayleigh-Ritz approximation techniques. The transient response is investigated for different device densities and different excitation voltages.

## 2. PROBLEM STATEMENT

Figure (1) shows a freely vibrating cantilever beam having four rows of uniformly spaced embedded micro-devices. Half of these devices are sensors and the other half are actuators. Every sensor is assumed to be connected to an actuator symmetrically placed on the opposite side of the beam neutral axis, through a proper electronic feedback loop. For the sake of simplicity, in the analysis we assume that two rows contain all sensors and the opposite rows contain all actuators. As the beam flexes, every sensor senses the external strain and its output is used in a closed-loop feedback circuit to actuate the corresponding actuator on the opposite side of the neutral axis. If the actuation strain is assumed to be proportional to the bending strain seen by the sensors, then the result is an apparent stiffening of the structure and an accompanying increase in the natural frequency, (all losses in the system are ignored) [Dasgupta and Alghamdi, 1992]. If the actuation strain is made proportional to the bending strain rate, then the result is an apparent damping of the structure, as illustrated in this study.

The aim of this paper is to study the transient behavior of the adaptive beam by assuming the feedback to the actuators to be proportional to the strain rate measured by the sensors. The damping characteristics of the beam at different device densities and different loading conditions are investigated. The analytical study uses Eshelby's equivalent inclusion technique and linear piezoelectric constitutive model, and is simplified significantly by several assumptions;

- I)- Euler-Bernoulli beam theory is applicable.
- II)- Each embedded micro-device is assumed to be a piezoelectric micro-cylinder of elliptical cross-section, whose polarization axis is oriented perpendicular to the length of the beam.
- III)- The length scale of each device is small compared to the beam dimensions.
- IV)- The bending strain is assumed to be uniform over the length scale of the device. This approximation greatly simplifies the algebra of the eigenstrain solution, without significant loss of accuracy [Alghamdi and Dasgupta, 1993c].
- V)- Each device is assumed to be embedded far enough below the free surface of the beam such that Eshelby's eigenstrain solution for infinite domains is applicable.
- VI)- The distance between neighboring devices is assumed to be large enough to prevent mutual interactions. Finally,
- VII)- The electrical field in each actuator is assumed to be uniform, as a first order approximation.

As a result of the above assumptions, each micro-device is approximated to act like an electro-elastic heterogeneity embedded in a large host structure. Perfect bonding is assumed at the interfaces between the devices and the host. Host and device materials are approximated to be linear and mechanically isotropic. All material properties are listed in Table (1). The linearizing assumption limits the validity of this approximate analysis to small far-field strains and small electrical potentials.

### 3. ANALYSIS

Eshelby's classical equivalent-inclusion technique is used to obtain the elastic interaction between the device and the host, under both external and internal applied loads [Eshelby, 1957]. External bending loads,  $\bar{\epsilon}^o$ , are handled through equivalent inclusion techniques as a fictitious, stress-free eigenstrain.

The internal actuation loads,  $\bar{\epsilon}^r$ , are treated as real eigenstrains and are obtained from the linearized, isothermal, coupled electro-mechanical constitutive model given below. In the present analytical context, sensors have only a fictitious eigenstrain due to external loads (neglecting any potential caused by direct effect), while actuators have both fictitious and real eigenstrains.

The linearized, isothermal, coupled electro-mechanical constitutive model is [Ikeda, 1990]:

$$\begin{aligned}\bar{\sigma} &= \underline{C} \bar{\epsilon} - \underline{h}^T \bar{E} \\ \bar{D} &= \underline{h} \bar{\epsilon} + \underline{e} \bar{E}\end{aligned}\tag{1}$$

where the mechanical stress,  $\bar{\sigma}$ , and the total strain,  $\bar{\epsilon}$ , are expressed as vector quantities. As an example, the stress vector  $\bar{\sigma}$  is  $[\sigma_{xx} \ \sigma_{yy} \ \sigma_{zz} \ \tau_{yz} \ \tau_{xz} \ \tau_{xy}]$ . The total strain  $\bar{\epsilon}$  includes mechanical (far-field and disturbance loadings) as well as electro-mechanical (induced) contributions.  $\bar{E}$  and  $\bar{D}$  are the electrical field and displacement vectors, respectively.  $\underline{C}$  is the mechanical stiffness matrix,  $\underline{h}$  is the piezoelectric coupling matrix indicating the stress caused by completely constrained excitation under a unit applied electrical field, and  $\underline{e}$  is the fully constrained dielectric matrix. Vector quantities are denoted by arrows, while an underscore is used to denote matrix quantities.

Eshelby's method for modeling the elastic interaction is based upon modeling the heterogeneous problem as an equivalent inclusion in homogeneous media with a fictitious eigenstrain which has the same stress field as the original heterogeneity, under both external loads and internal actuation loads. Thus, in the heterogeneity:

$$\begin{aligned}\bar{\sigma}^o + \bar{\sigma}' &= \underline{C}^D (\bar{\epsilon}^o + \bar{\epsilon}' - \bar{\epsilon}^r) \\ &= \underline{C}^H (\bar{\epsilon}^o + \bar{\epsilon}' - \bar{\epsilon}^*)\end{aligned}\tag{2}$$

where  $\bar{\epsilon}^* = \bar{\epsilon}^r + \bar{\epsilon}^f$ ; superscripts D and H indicate the device and the host, respectively; superscripts o, /, r, f, and \* on the stress and strain terms indicate applied far-field value, disturbance due to the presence of the heterogeneity, real actuation eigenstrains, fictitious eigenstrains due to external loading, and total eigenstrains, respectively. The real actuation eigenstrain is obtained from Equation (1) as:

$$\begin{aligned}\bar{\epsilon}^r &= \underline{d}^T \bar{E} \\ \text{where } \underline{d} &= \underline{h} \underline{S}^D\end{aligned}\tag{3}$$

$\underline{d}$  represents the free-expansion electromechanical matrix of the piezoelectric material for a unit applied

electric field and  $\underline{S}^D$  is compliance tensor of the device material.

The total eigenstrain is now related to the disturbance strain by Eshelby's strain concentration tensor  $\underline{S}^E$ :

$$\bar{\epsilon}^i = \underline{S}^E \bar{\epsilon}^* = \underline{S}^E (\bar{\epsilon}^r + \bar{\epsilon}^f) \quad (4)$$

Explicit forms for Eshelby's tensor are readily available in the literature for embedded isotropic heterogeneities of ellipsoidal geometries [Eshelby, 1957].

Substituting Equation (4) in Equation (2), we obtain:

$$\begin{aligned} \underline{C}^D [\bar{\epsilon}^o + \underline{S}^E (\bar{\epsilon}^r + \bar{\epsilon}^f) - \bar{\epsilon}^r] = \\ \underline{C}^H [\bar{\epsilon}^o + \underline{S}^E (\bar{\epsilon}^r + \bar{\epsilon}^f) - \bar{\epsilon}^f - \bar{\epsilon}^r] \end{aligned} \quad (5)$$

Equation (5) can now be solved for the unknown fictitious eigenstrain  $\bar{\epsilon}^f$  in terms of the applied external strain  $\bar{\epsilon}^o$  and the real actuation eigenstrain  $\bar{\epsilon}^r$ . Because the applied strains are assumed to be approximately uniform in this study, so are the fictitious eigenstrain, the disturbance strain, and the total strain. The fictitious eigenstrain can be written symbolically as

$$\bar{\epsilon}^f = \underline{A}^o \bar{\epsilon}^o + \underline{A}^r \bar{\epsilon}^r \quad (6)$$

where matrices  $\underline{A}^o$  and  $\underline{A}^r$  are functions of both host and device materials and the geometry of devices. Detailed expressions for  $\underline{A}^o$  and  $\underline{A}^r$  are omitted here for brevity.

As in Equation (5), the total strain is the summation of the far-field strain and the disturbance strain,

$$\bar{\epsilon} = \underline{G}^o \bar{\epsilon}^o + \underline{G}^r \bar{\epsilon}^r \quad (7)$$

where

$$\underline{G}^o = \underline{I} + \underline{S}^E \underline{A}^o, \text{ and } \underline{G}^r = \underline{S}^E \underline{A}^r.$$

$\underline{I}$  is the 6 by 6 identity matrix.

Now, the mechanical, electro-mechanical, and electrical energy of the structure is computed to obtain the transient response of the structure, through a suitable variational scheme. The variational principle is a generalized form of Hamilton's principle, and can be written as [Tiersten, 1967];

$$\delta \left[ \int_{t_0}^{t_1} (L + W) dt \right] = 0 \quad (8)$$

where the Lagrangian functional  $L$  is the difference between the kinetic energy  $T$  and the electric enthalpy  $H$ . The work term,  $W$ , includes all types work resulting from mechanical loadings and electrical charges.

Thus, using the definition of electric enthalpy [Tiersten, 1967] and neglecting all mechanical boundary terms in this free-vibration problem,  $L + W$  is given by,

$$L+W = \left(\frac{1}{2}\right) \int_V [\rho \dot{\bar{u}}^T \dot{\bar{u}} - \bar{\epsilon}^T \underline{C} \bar{\epsilon} + \bar{E}^T \underline{\epsilon} \bar{E} + 2 \bar{\epsilon}^T \underline{h}^T \bar{E}] dV - \int_S \phi \bar{n}^T \bar{D} dS \quad (9)$$

where  $\rho$  is the material density of host or device,  $\bar{u}$  is the displacement field,  $\phi$  is the electric potential,  $S$  is the surface area of the actuators, and  $\bar{n}$  is the unit outward normal vector on the surface of the devices.

The second integral in this equation is the mechanical energy term and is denoted as  $U_{mech}$ . Using Eshelby's analysis,  $U_{mech}$  is obtained as [Alghamdi and Dasgupta, 1993b],

$$2 U_{mech} = \int_V \bar{\epsilon}^{oT} \underline{C}^H \bar{\epsilon}^o dV + \int_{\Omega} \bar{\epsilon}^{*T} \underline{C}^H \underline{S}^E \bar{\epsilon}^* dV + \int_{\Omega} \bar{\epsilon}^{oT} \underline{\Delta C} \bar{\epsilon}^o dV + 2 \int_{\Omega} \bar{\epsilon}^{oT} \underline{\Delta C} \underline{S}^E \bar{\epsilon}^* dV + \int_{\Omega} \bar{\epsilon}^{*T} \underline{S}^{ET} \underline{\Delta C} \underline{S}^E \bar{\epsilon}^* dV \quad (10)$$

where  $\Omega$  is the volume of the devices, and  $\underline{\Delta C} = \underline{C}^D - \underline{C}^H$ .

The variation of  $(L+W)$  yields:

$$\int_{t_0}^t \left[ \int_V [\rho \dot{\bar{u}}^T \delta \dot{\bar{u}} - \bar{\epsilon}^T \underline{C} \delta \bar{\epsilon} + \bar{E}^T \underline{h} \delta \bar{\epsilon} + \delta \bar{E}^T \underline{h} \bar{\epsilon} + \bar{E}^T \underline{\epsilon} \delta \bar{E}] dV - \int_S [\delta \phi \bar{n}^T \bar{D} + \phi \bar{n}^T \delta \bar{D}] dS \right] dt = 0 \quad (11)$$

The first term in Equation (11) can be integrated by parts in time,

$$\int_{t_0}^t \int_V \rho \dot{\bar{u}}^T \delta \dot{\bar{u}} dV dt = - \int_{t_0}^t \int_V \rho \bar{u}^T \delta \bar{u} dV dt \quad (12)$$

Assuming that the displacement and electrical field functions are separable in space and time:

$$\begin{aligned} \bar{u}(\bar{x}, t) &= \bar{u}^o(\bar{x}) r(t) \\ \bar{E}(\bar{x}, t) &= \bar{E}^o(\bar{x}) v(t) \end{aligned} \quad (13)$$

Here,  $r(t)$  and  $v(t)$  may be considered to be generalized coordinates at any instant of time, with  $\bar{u}^o$  and  $\bar{E}^o$  acting as spatial interpolation functions.  $\bar{x}$  is the position vector. As shown in Equation (13), only one generalized coordinate is used to model each of the displacement and electrical variables in this problem.  $r(t)$  indicates the tip displacement of the cantilever beam and  $v(t)$  is a measure of the electrical potentials applied to actuate the beam.

Equations (10,12,13) are substituted into Equation (11) and all variables in Equation (11) are expressed

in terms of  $\bar{u}$  and  $\bar{E}$ . Allowing arbitrary variations of  $r(t)$  and  $v(t)$ , two equations are obtained in generalized coordinates. The first is the actuator equation, the second is the sensor equation [Hagood et al, 1990],

$$\begin{aligned} M \ddot{r}(t) + C^* \dot{r}(t) + C_a v(t) + K_a r(t) &= 0 && \text{Actuator Equation} \\ C_s v(t) + K_s r(t) &= 0 && \text{Sensor Equation} \end{aligned} \quad (14)$$

Where  $M$  is the mass, given as:

$$M = \int_V \rho \bar{u}^{oT} \bar{u}^o dV \quad (15)$$

The structural damping, denoted by  $C^*$ , is assumed to be proportional to  $M$  and  $K_a$  (Rayleigh damping), and is expressed as:

$$C^* = a M + b K_a$$

where  $a$  and  $b$  are material constants that can be determined from experiments. The remaining coefficient in Equation (14) are given as:

$$\begin{aligned} C_a = & \int_{\Omega} \bar{A}^{oT} \bar{\epsilon}^{oT} \bar{C}^H \bar{S}^E \bar{A}^r \bar{\epsilon}^r dV + \int_{\Omega} \bar{A}^{oT} \bar{\epsilon}^{oT} \bar{S}^{ET} \Delta \bar{C} \bar{S}^E \bar{A}^r \bar{\epsilon}^r dV \\ & + \int_{\Omega} \bar{A}^{oT} \bar{\epsilon}^{oT} \Delta \bar{C} \bar{S}^E \bar{A}^r \bar{\epsilon}^r dV - \int_{\Omega} \bar{E}^{oT} h \bar{G}^o \bar{\epsilon}^o dV + \int_S \bar{E}^{oT} h \bar{G}^o \bar{\epsilon}^o dS \end{aligned} \quad (16)$$

$$\begin{aligned} K_a = & \int_V \bar{\epsilon}^{oT} \bar{C}^H \bar{\epsilon}^o dV + \int_{\Omega} \bar{A}^{oT} \bar{\epsilon}^{oT} \bar{C}^H \bar{S}^E \bar{A}^o \bar{\epsilon}^o dV + \int_{\Omega} \bar{\epsilon}^{oT} \Delta \bar{C} \bar{\epsilon}^o dV \\ & + \int_{\Omega} \bar{A}^{oT} \bar{\epsilon}^{oT} \bar{S}^{ET} \Delta \bar{C} \bar{S}^E \bar{A}^o \bar{\epsilon}^o dV + 2 \int_{\Omega} \bar{A}^{oT} \bar{\epsilon}^{oT} \Delta \bar{C} \bar{S}^E \bar{A}^o \bar{\epsilon}^o dV \end{aligned} \quad (17)$$

$$\begin{aligned} K_s = & \int_{\Omega} \bar{A}^{rT} \bar{\epsilon}^{rT} \bar{C}^H \bar{S}^E \bar{A}^o \bar{\epsilon}^o dV + \int_{\Omega} \bar{A}^{rT} \bar{\epsilon}^{rT} \bar{S}^{ET} \Delta \bar{C} \bar{S}^E \bar{A}^o \bar{\epsilon}^o dV \\ & + \int_{\Omega} \bar{A}^{rT} \bar{\epsilon}^{rT} \bar{S}^{ET} \Delta \bar{C} \bar{A}^o \bar{\epsilon}^o dV - \int_{\Omega} \bar{E}^{oT} h \bar{G}^o \bar{\epsilon}^o dV + \int_S \bar{E}^{oT} h \bar{G}^o \bar{\epsilon}^o dS \end{aligned} \quad (18)$$

$$\begin{aligned} C_s = & \int_{\Omega} \bar{A}^{rT} \bar{\epsilon}^{rT} \bar{C}^H \bar{S}^E \bar{A}^r \bar{\epsilon}^r dV + \int_{\Omega} \bar{A}^{rT} \bar{\epsilon}^{rT} \bar{S}^{ET} \Delta \bar{C} \bar{S}^E \bar{A}^r \bar{\epsilon}^r dV \\ & + 2 \int_S \bar{E}^{oT} h \bar{G}^r \bar{\epsilon}^r dS - 2 \int_{\Omega} \bar{E}^{oT} h \bar{G}^r \bar{\epsilon}^r dV \\ & + 2 \int_S \bar{E}^{oT} \underline{\epsilon} \bar{E}^o dS - \int_V \bar{E}^{oT} \underline{\epsilon} \bar{E}^o dV \end{aligned} \quad (19)$$

If  $v(t)$  is assumed to be proportional to  $r(t)$ , we obtain an apparent stiffening [Alghamdi and Dasgupta, 1993a,b], and if we assume  $v(t)$  to be proportional to  $\dot{r}(t)$ , we obtain an apparent damping, as shown below:

$$M \ddot{r}(t) + [C^* + C_a] \dot{r}(t) + K_a r(t) = 0 \quad (20)$$

#### 4. RAYLEIGH-RITZ APPROXIMATION

In order to perform the integrations in Equations (15-19), all that remains now is to assume explicit representations for the applied flexural strain field, and the actuation eigenstrain. In this example, an approximate sinusoidal displacement field is assumed. Thus, the transverse deformation of the beam neutral axis is given as;

$$w(\bar{x}, t) = \sum_1^n r_n(t) \left[ 1 - \cos\left(\frac{n\pi y}{2L}\right) \right] \quad (21)$$

where the  $y$  axis is oriented along the length of the beam,  $w$  is the transverse displacement in the  $z$  direction, and  $r_n(t)$  is the tip displacement in the  $n^{\text{th}}$  mode,  $L$  is the length of the beam, and  $t$  is time. Assuming only the fundamental mode ( $n = 1$ ), and using the Euler-Bernoulli assumptions, the only non-zero term in the bending strain field  $\bar{\epsilon}^o$  is  $\epsilon_2^o$ , and is given as:

$$\epsilon_2^o(\bar{x}, t) = z \frac{\pi^2}{4L^2} r_1(t) \cos\left(\frac{\pi y}{2L}\right) \quad (22)$$

where,  $z$  is the distance of the micro-device from the neutral axis of the beam.

The only non-zero component of the actuation voltage vector is now  $E_3$  and is assumed to be proportional to the time-derivative of the output of the sensory devices, and hence, to the bending strain rate. In other words  $v(t) = K \dot{r}(t)$  where  $K$  is a proportionality constant, termed the gain factor in this study. Thus  $E_3$  is written as:

$$E_3(\bar{x}, t) = K z_i \frac{\pi^2}{4L^2} \dot{r}(t) \cos\left(\frac{\pi y_i}{2L}\right) \quad (23)$$

where  $z_i$  and  $y_i$  are the coordinates of the center of each device. The non-zero terms of the actuation strain vector are now written as:

$$\epsilon_i^x = d_{3i} E_3, \quad i = 1-6 \quad (24)$$

Equations (21-24) are used in Equations (15-19) to compute all terms in Equation (20). Solutions for Equation (20) are readily available in the literature.

## 5. RESULTS AND DISCUSSION

Figure (2a) shows the transient response of the adaptive beam having 24 actuators with volume fraction equal to 12% of the host. For simplicity, we assume in this illustrative example that the higher order mode shapes can be omitted. The tip amplitude is normalized by the initial condition at time  $t=0$ . Initially, the beam is allowed to vibrate under its own structural passive damping. Then at time equal to 0.5 second, the control circuit is activated by imposing a maximum electrical field,  $E_1$ , equal to 100 V/m. The applied electrical field to the actuators near the fixed end have the maximum value. The applied electrical potential is proportional to the time-rate of the sensory output. As a result of this activation, the system starts to damp faster, i.e. at a higher rate. The dashed line in Figure (2a) represents the decay envelop connecting all the peaks. In Figure (2b), the volume fraction ( $V_p$ ) of devices has been varied from 0 to 12% by increasing the number of devices from 0 to 24. The figure shows decay envelopes for 0, 2, 6, and 12% volume fractions. As expected, the damping increases with the increase in the device density.

Figure (3) shows the system response for a constant number of devices, but for different applied electrical field strengths. As the applied potential increases, the system damps faster. For convenience, the actuation strain amplitude ( $\epsilon^x$ ) is normalized with respect to the far-field strain amplitude ( $\epsilon^0$ ).

The damping time  $t_d$  of the system is quantified arbitrarily as the time for the amplitude to decrease to 1% of initial condition. The time  $t_d$  for active damping is then normalized for convenience, by the time  $t_d^0$  for passive damping alone (zero actuation strain). Figure (4) illustrates the effect of increasing the volume fraction of devices on the required normalized time to damp,  $t_d$ . As the volume fraction increases, the time to damp decreases. Clearly, as the applied excitation strains increase the time to damp decreases.

The location of actuators with respect to the beam mid-plane is a very influential parameter. The bending strain (and hence, the actuation strain) increases as we move both the sensors and actuators away from the mid-plane. The dependence of the damping effect (time to damp) on the location of the actuators, relative to the neutral axis, is shown in Figure (5). As expected, moving the actuators away from the neutral axis decreases the required time to damp.

## 6. CONCLUSIONS

This paper discusses the use of Eshelby's equivalent inclusion techniques for modeling the interaction between micro-devices and the surrounding host in active vibration damping applications. This method presents a unified approach for addressing the interaction mechanics of micro-devices embedded in an adaptive structure. This method is integrated with the linear piezoelectric constitutive model and Hamilton's principle to formulate the system dynamic equation for transient response behavior of an adaptive beam. Preliminary results, obtained by Rayleigh-Ritz approximate method, show excellent capability of modeling the transient response by Eshelby's method.

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	E (GPa)	$\nu$	$d_{31}$	$d_{33}$	$d_{15}$	$\epsilon_{11}$	$\epsilon_{33}$
			(C/N) * 1e-12			(C/mV) * 1e-10	
PZT-5H	64	0.39	-274	593	741	150	130
ALPLEX	1.4	0.30	—	—	—	—	—

TABLE 1. ELECTROMECHANICAL PROPERTIES OF DEVICE AND HOST MATERIALS.

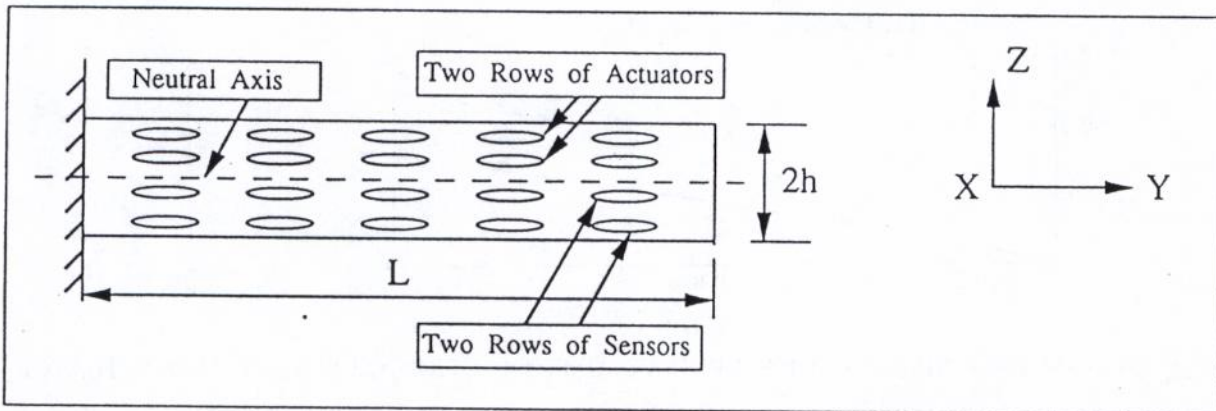


FIGURE 1. ADAPTIVE BEAM WITH EMBEDDED ROWS OF MICRO-DEVICES.

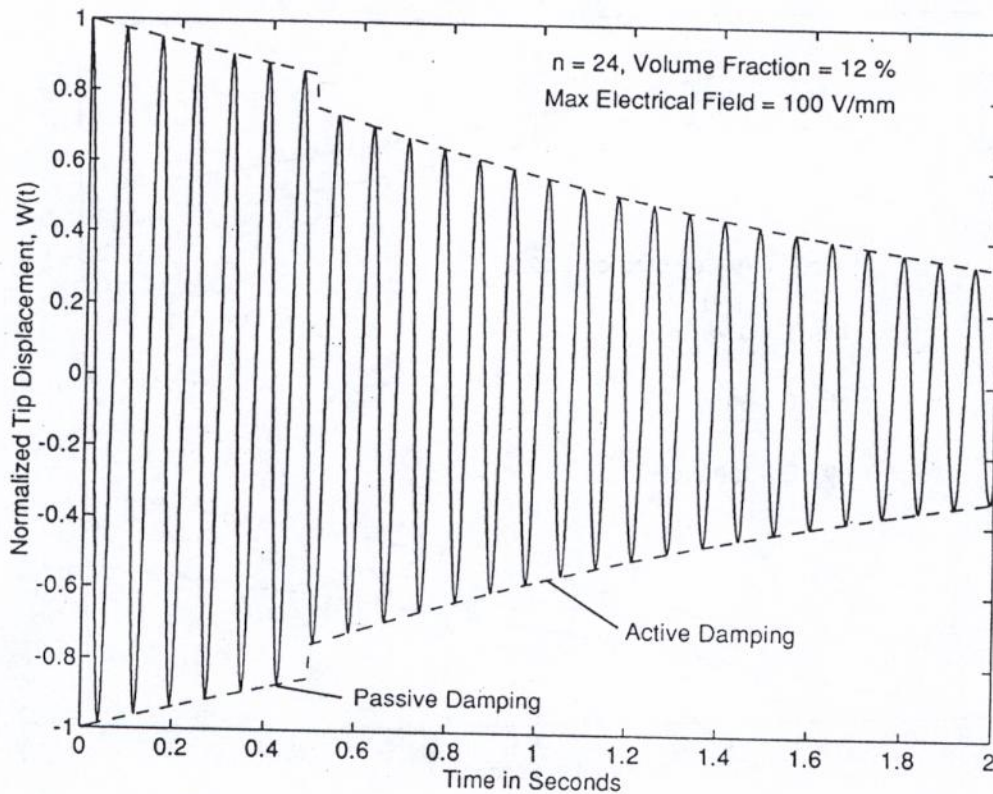


FIGURE 2a. SYSTEM RESPONSE FOR 12% VOLUME FRACTION OF SENSOR AND ACTUATOR DEVICES.

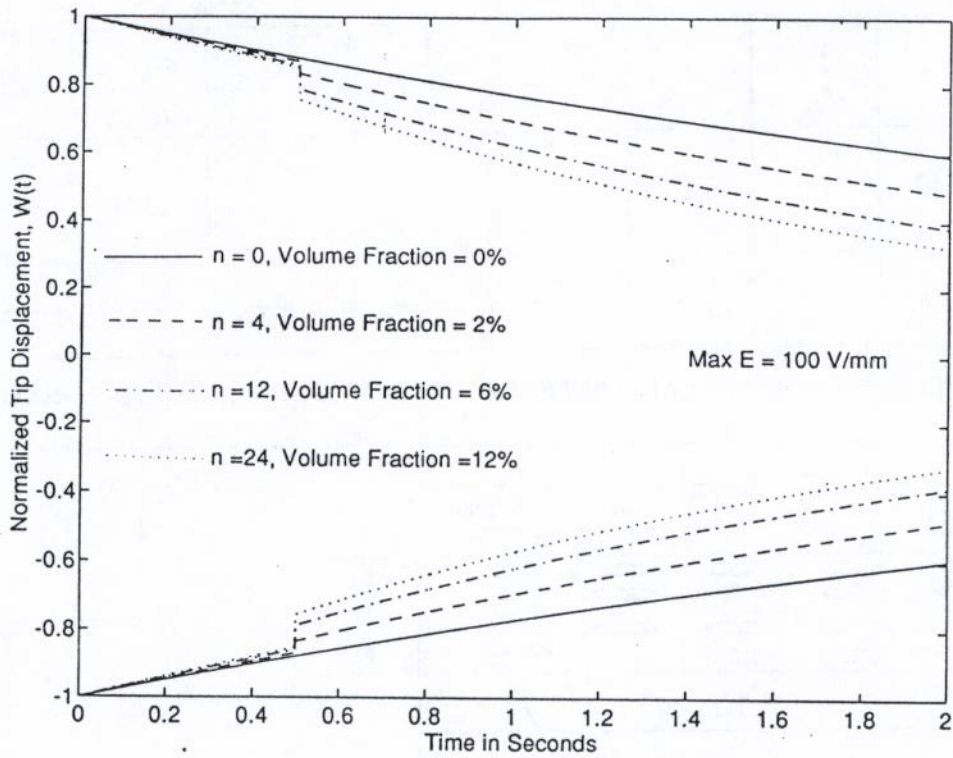


FIGURE 2b. SYSTEM RESPONSE FOR DIFFERENT VOLUME FRACTION OF SENSOR AND ACTUATOR DEVICES.

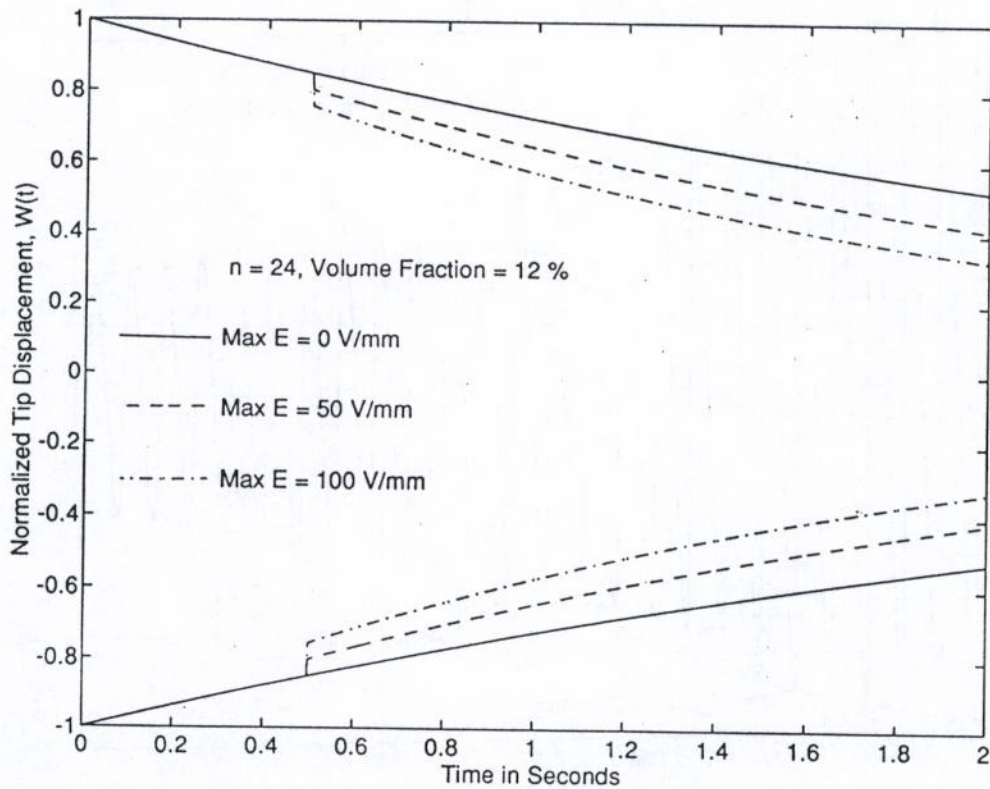


FIGURE 3. SYSTEM RESPONSE FOR DIFFERENT APPLIED ELECTRICAL FIELD STRENGTHS.

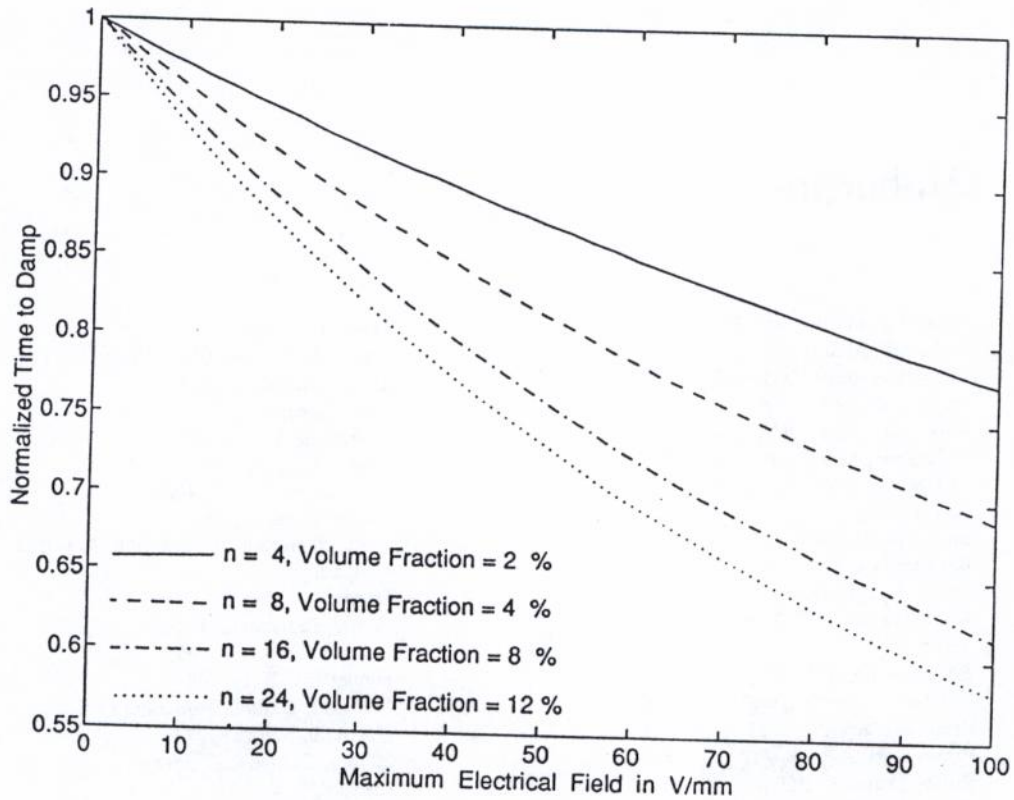


FIGURE 4. TIME TO DAMP VS. ACTUATION STRAIN (FIELD STRENGTH) FOR SEVERAL DFVICES DENSITIES.

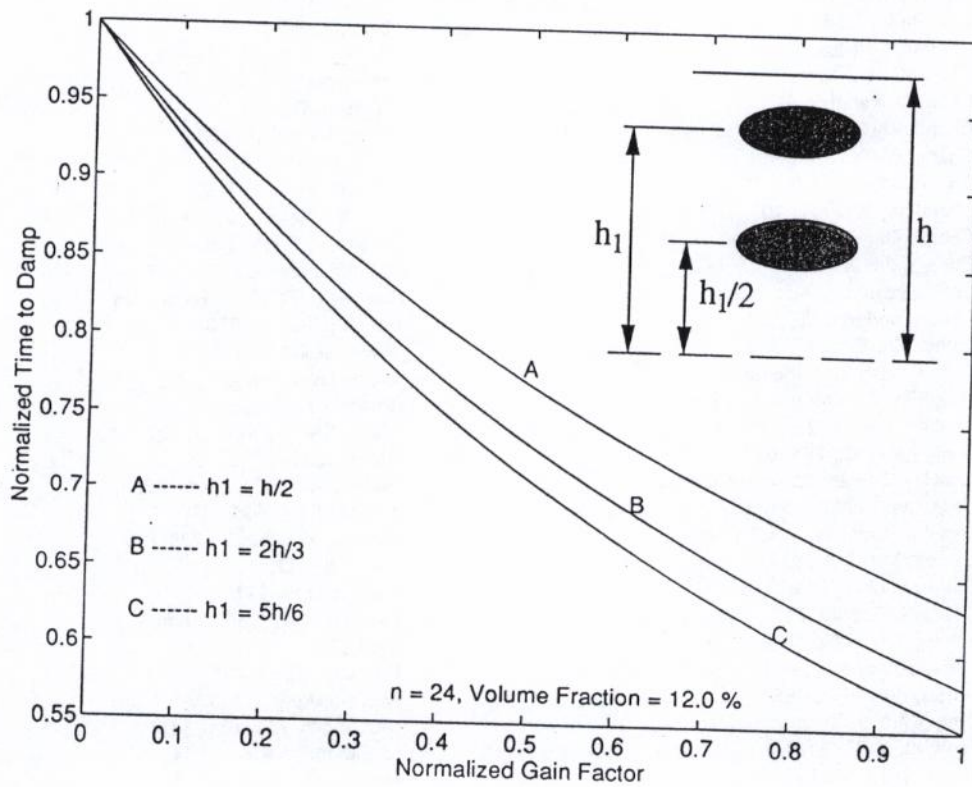


FIGURE 5. THE EFFECT OF DEVICE POSITION ON THE DAMPING.

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